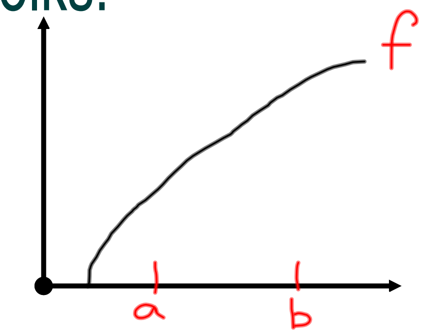
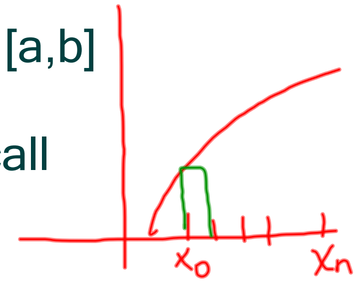
The Definite Integral for a general function f

Here it finally is folks!



1) Let f be a continuous bounded function on [a,b]



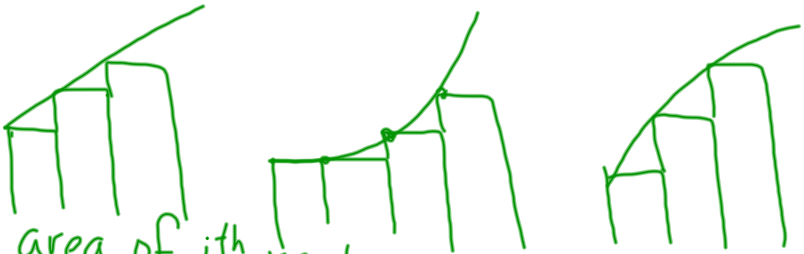
2) Subdivide [a,b] into n equal subintervals (call points $\underbrace{x_0}_{a}, x_1, x_2, x_3, x_4 \ldots x_{n-1}, \underbrace{x_n}_{b}$

3) What is the length of each subinterval?

$$\Delta x = \frac{b-a}{n}$$

4) Construct the left hand sum...that over or under approximates area under the curve here?

because the curve is increasing.

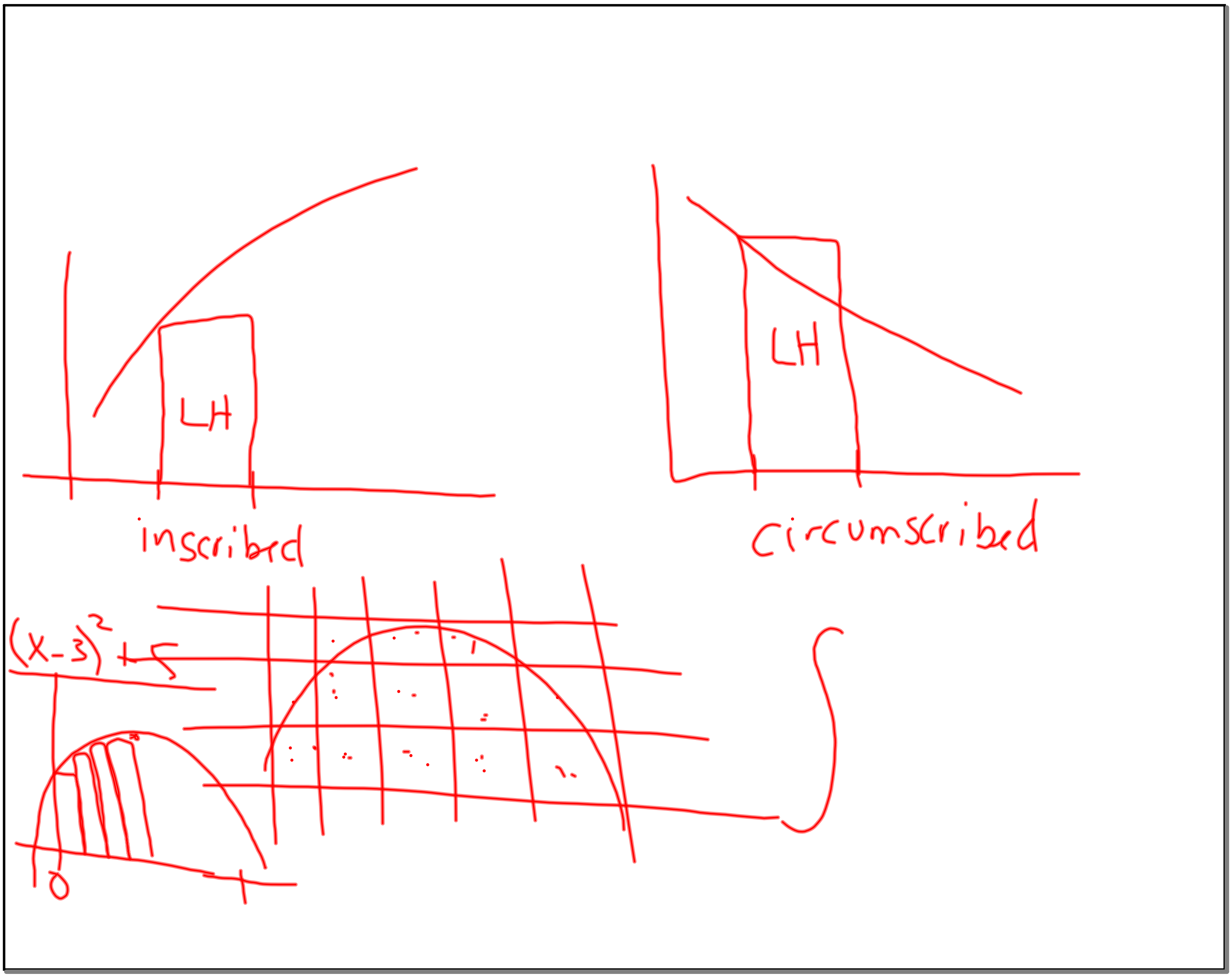


area of i^{th} rect

$f(x_i)\Delta x$

$$\sum_{i=0}^{n-1} f(x_i)\Delta x$$

Riemann Sum


LH
inscribed
LH
circumscribed
(x-3)^2+5
0
1

4) cont. 

LEFT HAND SUM:

5) Construct the RIGHT HAND SUM which over/under approximates the area under the curve here...

$$\sum_{i=1}^{n} \underbrace{f(x_i)}_{\text{height of the } i^{th} \text{ rectangle}} \overbrace{\Delta x}^{\text{width of the } i^{th}}$$

6) Now, refine the partition...how so?

make n get $\underline{\text{HUGE}}$, i.e. $n \to \infty$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \; [a,b] = \lim_{n\to\infty} \sum_{i=0}^{n-1} f(x_i)\Delta x \; [a,b]$$

$$= \int_a^b f(x)\,dx$$

THIS is the mathematical definition of the DEFINITE INTEGRAL!!!!

$$\int_a^b f(x)\,dx = \lim_{\substack{\Delta x \to 0 \\ n\to\infty}} \sum_{\substack{i=1 \\ [a,b]}}^{?} f(x_i)\Delta x = \lim_{\substack{\Delta t \to 0 \\ n\to\infty}} \sum_{\substack{i=0 \\ [a,b]}}^{?} f(t_i)\Delta t$$

What are some $\boxed{\text{interpretations}}$?

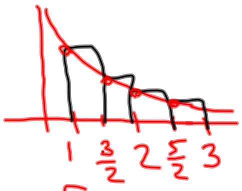
velocity * time = distance

$\int_a^b v(t)\,dt$ = displacement of object from time a to time b.

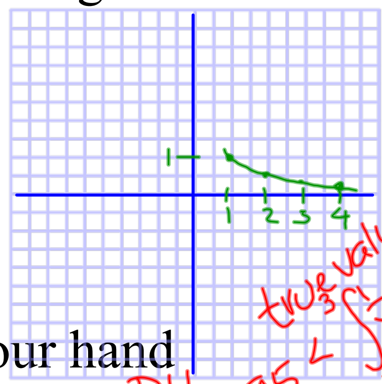
$f(x)\cdot\Delta x$ =net area bounded by the graph of f, the x axis and lines x=a, x=b

PRACTICE: $\int_1^3 \frac{1}{t}dt$ 

1) Use a left hand sum and a right hand sum with 4 subdivisions to approximate this integral.



$\frac{3-1}{4} = \frac{1}{2}$



$\frac{1}{2}\left[1 + \frac{2}{3} + \frac{1}{2} + \frac{2}{5}\right] = 1.283$

true value $.95 < \int_1^3 \frac{1}{x}dx < 1.283$

RH .95

Now, using sum seq, a) check your hand calculations above.

$\frac{1}{2}\,\text{sum}(\text{seq}(\frac{1}{x}, x, 1, 3-\frac{1}{2}, \frac{1}{2}))$

$\underbrace{\frac{1}{2}(\text{sum}(\text{seq}(\frac{1}{x}, x, 1.5, 3, \frac{1}{2}))}_{.95}$

b) use 10 subdivisions (LH and RH)

c) Use 100 subdivisions

LH 1.105 RH 1.091

$\frac{1}{50}\,\text{sum}(\text{seq}(\frac{1}{x}, x, 1, 3-\frac{1}{50}, \frac{1}{50}))$

$\frac{1}{50}\,\text{sum}(\text{seq}(\frac{1}{x}, x, 1+\frac{1}{50}, 3, \frac{1}{50}))$

d) Guess what you think the limit is.

≈ 1.098 ≈ 1.1

e) Use the integral command on your TI-89 to find it.

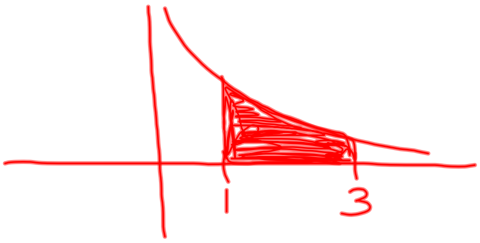
$$\int_1^3 \frac{1}{x}dx = \ln 3 = 1.098$$

$\int(1/x, x, 1, 3)$

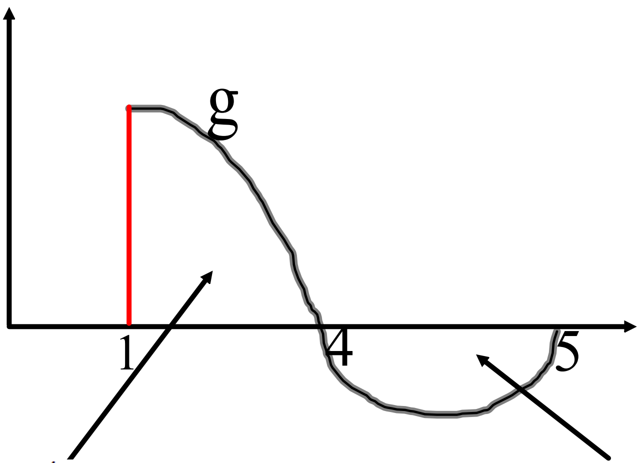
$$\int_1^3 \frac{1}{t}\,dt$$

1.098

As $n \to \infty$, the left hand and right hand sums converge toward the same *number*. What does this number represent?



2) Approximate $\int_0^{\frac{\pi}{2}} \cos(x)dx$ with 2, 10, 50, 200 subdivisions and state in words what this represents.



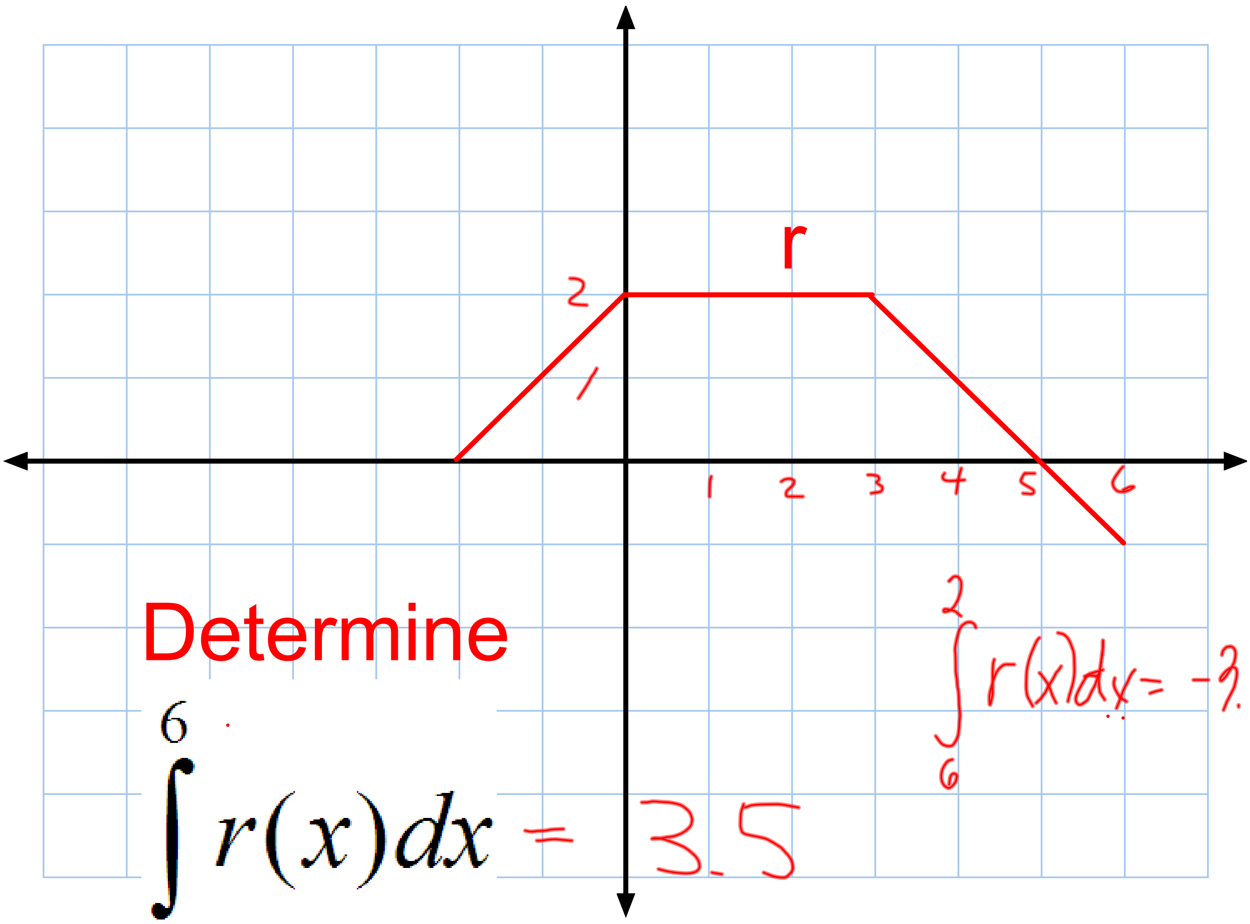
$$\int_5^2 f(x)\,dx$$

neg

Given that $\int_1^4 g(x)dx = 10$ **and** $\int_4^5 g(x)dx = -3$

neg.

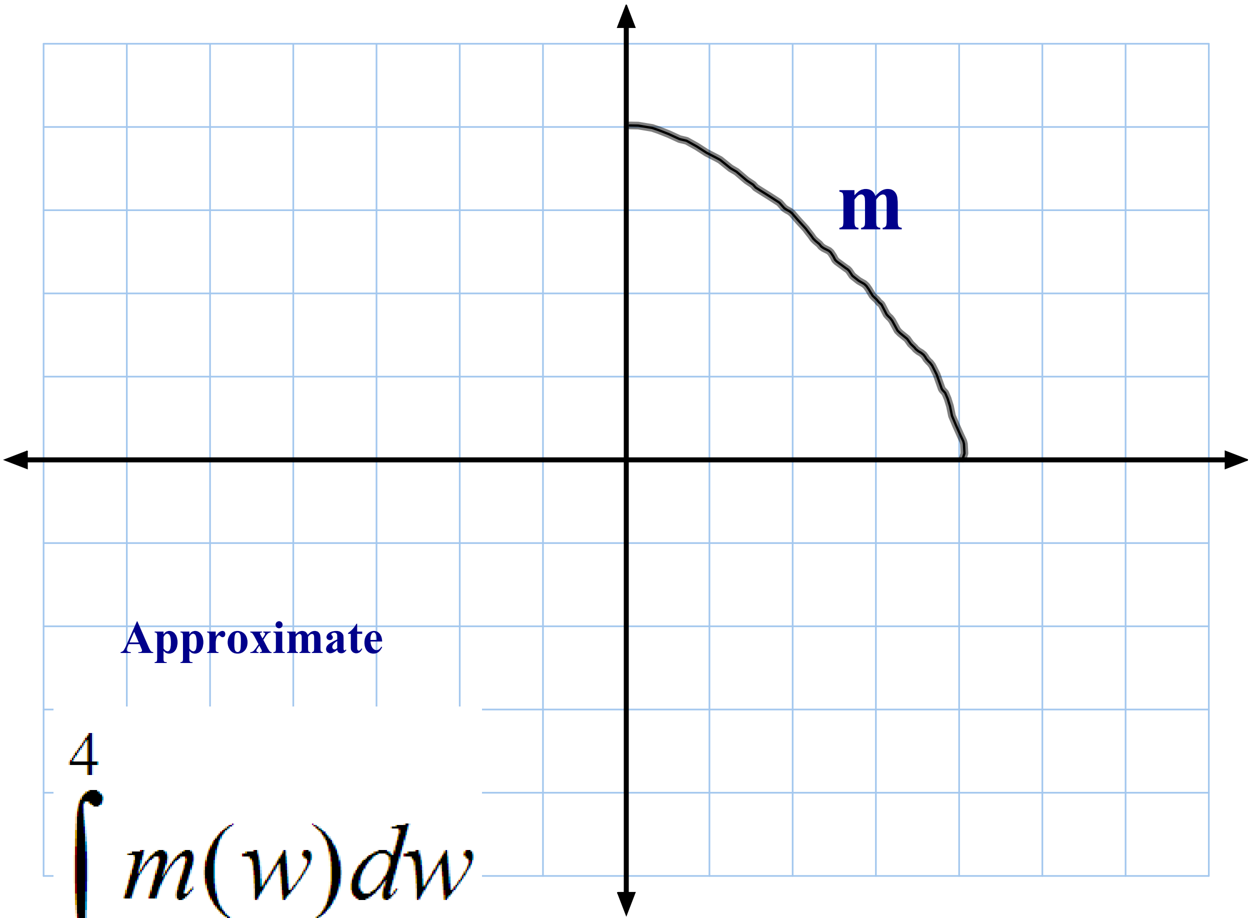
determine $\int_1^5 g(x)dx = 7 = \int_1^4 g(x)dx + \int_4^5 g(x)dx$



Determine

$$\int_2^6 r(x)\,dx = 3.5$$

$$\int_6^2 r(x)\,dx = -3.5$$



Approximate

$$\int_0^4 m(w)\,dw$$

3) Determine the area between f(x)= $x^{\frac{1}{2}}$ and g(x)= $x^{\frac{1}{3}}$ for $0 \le x \le 1$

4) Consider the graph of $f(x) = x^3 + x^2 - 6x$
Determine the total area between the graph and hte x axis between -3 and 2.

http://www.slu.edu/classes/maymk/Riemann/Riemann.html

http://science.kennesaw.edu/~plaval/tools/integration.html