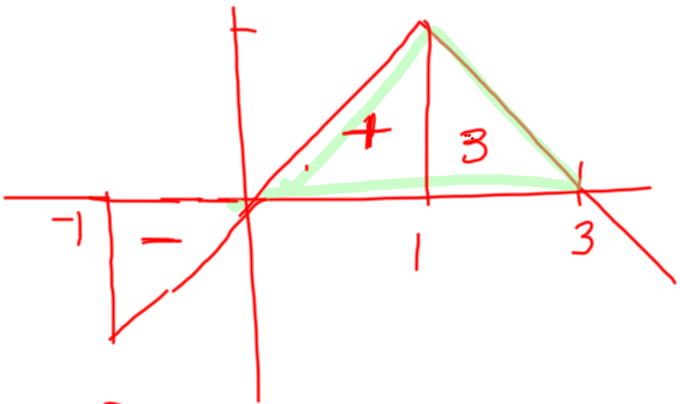
# THE PERSPECTIVES OF THE DEFINITE INTEGRAL PLUS ONE MORE APPLICATION OF IT....

**REVIEW.....** $\int_r^s g(y)\,dy$ **MEANS:**

**1)(math):** $\underset{\text{defn}}{=} \lim_{\substack{n\to\infty \\ [r,s]}} \sum_{i=0}^{n-1} g(y_i)\,\Delta y$

**2)(graph):** net area bounded by graph of $g$, the $x$-axis and the lines $y=r$ and $y=s$.

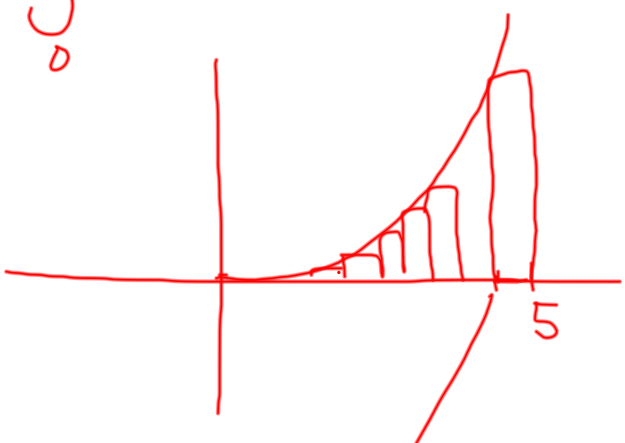
**3)(situation):**
Integrating a rate of a quantity results in the total change of the quantity over the interval.




$$\int_{-1}^{3} g(x)\,dx$$

$$\int_{0}^{5} x^2\,dx$$

$$\underbrace{\frac{5-0}{50}}_{\frac{1}{10}}$$

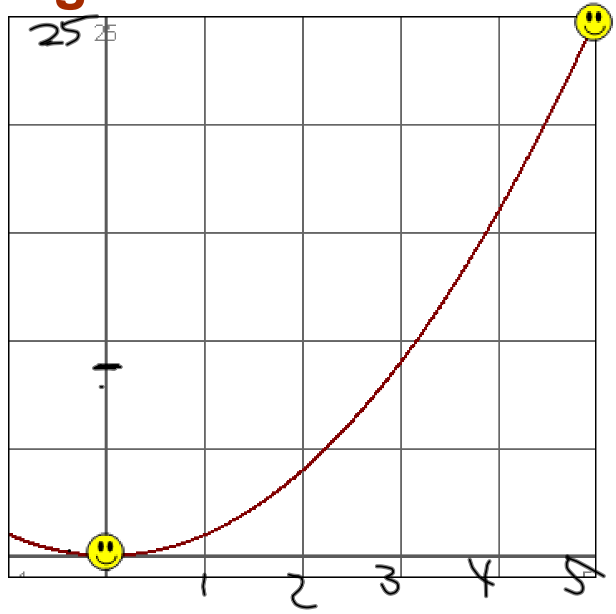



$$\frac{1}{10}\,\mathrm{sum}\left(\mathrm{seq}\left(x^2, x, 0, 5-\frac{1}{10}, \frac{1}{10}\right)\right) \quad 5-\frac{1}{10}$$

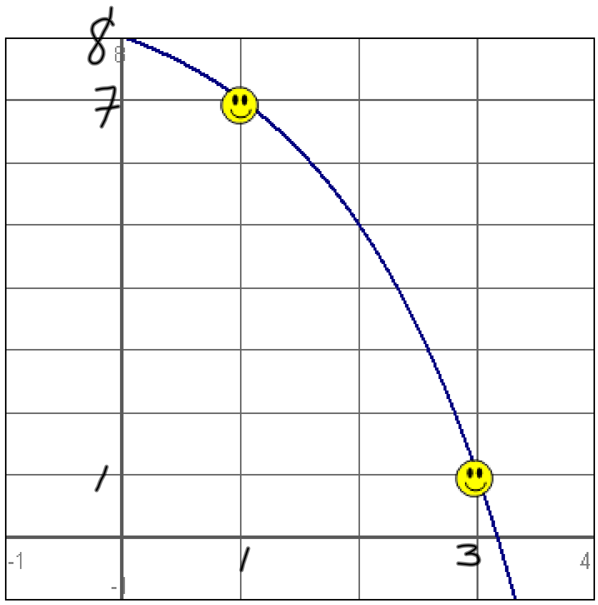
# HERE'S ANOTHER APPLICATION...

It can help us determine the AVERAGE FUNCTION VALUE...that's right! If we wanted to average ALL the function values (that's an infinite number of function values!) say, of $f(x)=x^2$ from $x=0$ to $x=5$, we could actually do it with the help of the integral.    First, let's guess what it would be close(r) to....

8
7



What about this graph?  What do you think is the approximate average function value for x between 1 and 3? (is it closer to 7 or 1?)



## Average Function Value...the development of the formula...

**How would you calculate the average of tests scores named $f(x_1)$, $f(x_2)$, $f(x_3)$,...$f(x_n)$?**

$$\frac{\sum_{i=1}^{n} f(x_i)}{n}$$

**In your response above, you should have an 'n' value but we know** $\Delta x =$ $\underline{\frac{b-a}{n}}$ **so, n=**$\underline{\frac{b-a}{\Delta x}}$**.Replace n with this value.**

$$\frac{\sum_{i=1}^{n} f(x_i)\Delta x}{b-a}$$

$$\Delta x = \frac{b-a}{n}$$

**Now, extend this to LOTS o scores...infinitely many...**

**Average of n scores:** $$\frac{\sum_{i=1}^{n} f(x_i)\Delta x}{b-a}$$

$$\lim_{n\to\infty} \frac{\sum_{i=1}^{n} f(x_i)\Delta x}{b-a}$$

**Now, extend this to LOTS of test scores...infinitely many...**

$$\lim_{n\to\infty} \frac{\sum_{i=1}^{n} f(x_i)\Delta x}{b-a} = \boxed{\frac{\int_a^b f(x)dx}{b-a} = \text{AFV}}$$

AFV

AVERAGE FUNCTION VALUE OF f over [a,b]

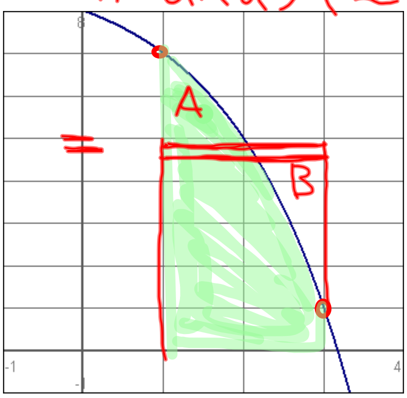
$$\frac{\int_0^5 x^2 dx}{5} = 8.3\overline{33}$$

**So, if** $\dfrac{\int_a^b f(x)dx}{b-a}$ = AFV AVERAGE FUNCTION VALUE OF f over [a,b]

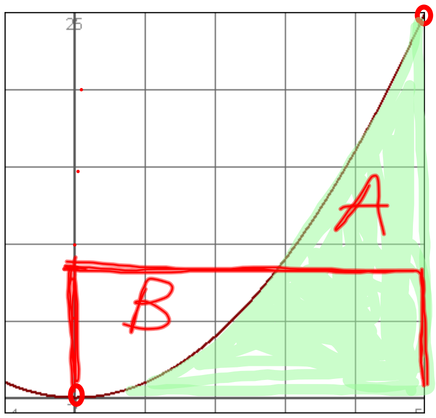
## What does this mean graphically?

$$\int_a^b f(x)dx = \underbrace{(AFV)(b-a)}_{\text{area of rectangle}}$$

To est. the AFV graphically, try to make a rect. such that area A ≈ area B.



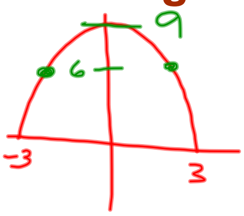

guess: 4.5~5 ish




we know: $8.\overline{3}$

# Examples:

1) An arch has the shape of $y=9-x^2$. What is the average height and where (x-wise) would you erect a support having that height?

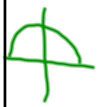



$$\frac{\int_{-3}^{3}(9-x^2)\,dx}{3--3} = AFV = 6$$

ave height

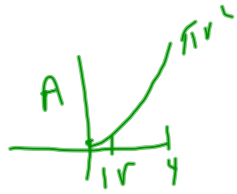
$6=9-x^2$ $\therefore x^2=3$ $x=\pm\sqrt{3}$

2) What is the average area of circles with radii (1,4)cm?



$\pi$ $16\pi$

$$\frac{\int_{1}^{4}\pi r^2\,dr}{4-1} = 7\pi$$




3) The hunger of sharks weighing x tons is $h(x)=x^3$. What is the average hunger of sharks weighing between 0 and 2 tons?

$$\frac{\int_{0}^{2}x^3\,dx}{2}$$

## Examples (cont).

**4) Given $s(t)=5t^3-3t^2$ where s is the position function over [1,2], determine the $\boxed{\text{average velocity}}$ two different ways.**

1 $$\frac{\int_1^2 v(t)\,dt}{2-1} = \frac{\int_1^2 (15t^2-6t)\,dt}{2-1}$$

2 $$\frac{s(2)-s(1)}{2-1}$$

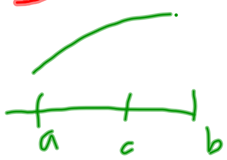
# Properties of Limits of Integration

If a, b and c are any numbers and f is a continuous function,

1) $$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

2) $$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

Does it have to be the case that $a<b<c$? no



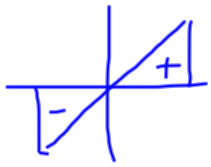
Let $f$ and $g$ be continuous functions and let $c$ be a constant...

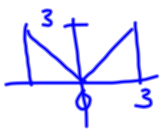
3) $\int_a^b [f(x) \pm g(x)]\,dx = \underline{\int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx}$

4) $\int_a^b c \cdot f(x)\,dx = \underline{c \int_a^b f(x)\,dx}$

Use symmetry to evaluate integrals when it simplifies your work...

ex $\displaystyle\int_{-3}^{3} x \, dx = 0$




ex $\displaystyle\int_{-3}^{3} |x| \, dx = 2\int_{0}^{3} x \, dx = \frac{1}{2} \cdot 3 \cdot 3 = \frac{9}{2}$