

6.2 Worksheet: Rational Exponents and Simplifying Radicals

1. Simplify each expression. Write your answer in exponential form a^n .

$$\begin{aligned} \text{a. } 9^{\frac{2}{3}} \cdot 27^{\frac{1}{2}} &= (3^2)^{\frac{2}{3}} \cdot (3^3)^{\frac{1}{2}} = 3^{\frac{4}{3}} \cdot 3^{\frac{3}{2}} \\ &= 3^{\frac{8}{6} + \frac{9}{6}} = \boxed{3^{\frac{17}{6}}} \end{aligned}$$

$$\begin{aligned} \text{b. } 25^{\frac{1}{4}} \cdot 125^{\frac{2}{5}} &= (5^2)^{\frac{1}{4}} \cdot (5^3)^{\frac{2}{5}} \\ &= 5^{\frac{1}{2}} \cdot 5^{\frac{6}{5}} = 5^{\frac{1}{2} + \frac{6}{5}} = 5^{\frac{13}{10}} = \boxed{5^{\frac{13}{10}}} \end{aligned}$$

$$\begin{aligned} \text{c. } \frac{3^{\frac{5}{2}}}{9^{\frac{2}{3}}} &= \frac{3^{\frac{5}{2}}}{(3^2)^{\frac{2}{3}}} = \frac{3^{\frac{5}{2}}}{3^{\frac{4}{3}}} = 3^{\frac{5}{2} - \frac{4}{3}} \\ &= 3^{\frac{15}{6} - \frac{8}{6}} = \boxed{3^{\frac{7}{6}}} \end{aligned}$$

$$\begin{aligned} \text{d. } \frac{625^{\frac{1}{3}}}{125^{\frac{2}{5}}} &= \frac{(5^4)^{\frac{1}{3}}}{(5^3)^{\frac{2}{5}}} = \frac{5^{\frac{4}{3}}}{5^{\frac{6}{5}}} = 5^{\frac{4}{3} - \frac{6}{5}} \\ &= 5^{-\frac{20}{15} + \frac{24}{15}} = \boxed{5^{\frac{4}{15}}} \end{aligned}$$

$$\begin{aligned} \text{e. } \frac{8^{\frac{1}{4}}}{4^{\frac{2}{3}}} &= \frac{(2^3)^{\frac{1}{4}}}{(2^2)^{\frac{2}{3}}} = \frac{2^{\frac{3}{4}}}{2^{\frac{4}{3}}} = 2^{\frac{3}{4} - \frac{4}{3}} \\ &= 2^{\frac{9}{12} - \frac{16}{12}} = \boxed{2^{-\frac{7}{12}}} \end{aligned}$$

$$\begin{aligned} \text{f. } \left(8^{\frac{3}{4}}\right)^{\frac{1}{2}} &= 8^{\frac{3}{8}} = 8^{\frac{3}{8}} \\ \text{or } \left((2^3)^{\frac{3}{4}}\right)^{\frac{1}{2}} &= (2^{\frac{9}{4}})^{\frac{1}{2}} = 2^{\frac{9}{8}} = \boxed{2^{\frac{9}{8}}} \end{aligned}$$

2. Rewrite each root as a power then simplify. Write your answer in exponential form a^n .

$$\begin{aligned} \text{a. } \sqrt{8} \cdot \sqrt[3]{32} &= 8^{\frac{1}{2}} \cdot 32^{\frac{1}{3}} = (2^3)^{\frac{1}{2}} \cdot (2^5)^{\frac{1}{3}} \\ &= 2^{\frac{3}{2}} \cdot 2^{\frac{5}{3}} = \boxed{2^{\frac{19}{6}}} \end{aligned}$$

$$\begin{aligned} \text{b. } \sqrt[3]{625} \cdot \sqrt[4]{5} &= 625^{\frac{1}{3}} \cdot 5^{\frac{1}{4}} = (5^4)^{\frac{1}{3}} \cdot 5^{\frac{1}{4}} \\ &= 5^{\frac{4}{3}} \cdot 5^{\frac{1}{4}} = 5^{\frac{16}{12} + \frac{3}{12}} = \boxed{5^{\frac{19}{12}}} \end{aligned}$$

$$\begin{aligned} \text{c. } \frac{\sqrt[3]{64}}{\sqrt[5]{16}} &= \frac{64^{\frac{1}{3}}}{16^{\frac{1}{5}}} = \frac{(4^3)^{\frac{1}{3}}}{(4^2)^{\frac{1}{5}}} = \frac{4^1}{4^{\frac{2}{5}}} \\ &= 4^{1 - \frac{2}{5}} = \boxed{4^{\frac{3}{5}}} \end{aligned}$$

$$\begin{aligned} \text{d. } \sqrt[3]{81} \cdot \sqrt[4]{9} &= 81^{\frac{1}{3}} \cdot 9^{\frac{1}{4}} = (3^4)^{\frac{1}{3}} \cdot (3^2)^{\frac{1}{4}} \\ &= 3^{\frac{4}{3}} \cdot 3^{\frac{1}{2}} = 3^{\frac{8}{6} + \frac{3}{6}} = \boxed{3^{\frac{11}{6}}} \end{aligned}$$

$$\begin{aligned} \text{e. } \frac{\sqrt[4]{27}}{\sqrt[5]{3}} &= \frac{27^{\frac{1}{4}}}{3^{\frac{1}{5}}} = \frac{(3^3)^{\frac{1}{4}}}{3^{\frac{1}{5}}} = \frac{3^{\frac{3}{4}}}{3^{\frac{1}{5}}} \\ &= 3^{\frac{3}{4} - \frac{1}{5}} = 3^{\frac{15-4}{20}} = \boxed{3^{\frac{11}{20}}} \end{aligned}$$

$$\begin{aligned} \text{f. } \frac{\sqrt[3]{4}}{\sqrt[5]{32}} &= \frac{4^{\frac{1}{3}}}{32^{\frac{1}{5}}} = \frac{(2^2)^{\frac{1}{3}}}{(2^5)^{\frac{1}{5}}} = \frac{2^{\frac{2}{3}}}{2^1} \\ &= 2^{\frac{2}{3} - 1} = 2^{-\frac{1}{3}} = \boxed{2^{-\frac{1}{3}}} \end{aligned}$$



3. Use a calculator to approximate each of the following to three decimal places.

a. $\sqrt{715} \quad \underline{26.739}$

b. $\sqrt[3]{917} \quad \underline{9.715}$

c. $\sqrt[3]{112} \quad \underline{3.253}$

d. $7\sqrt{51} - 8\sqrt{93} \quad \underline{-27.159}$

e. $(7\sqrt{5} - 3\sqrt{2})(8\sqrt{3} + \sqrt{7}) \quad \underline{188.29}$

f. $\frac{4\sqrt{5} - 7}{2\sqrt{3} + 8} \quad \underline{1.170} \quad \frac{1.9442719}{11.46410}$

4. Write the following in simplest radical form.

a. $\sqrt{320} = \sqrt{64 \cdot 5} = \underline{8\sqrt{5}}$

b. $\sqrt{136} = \sqrt[3]{8 \cdot 17} = \underline{2\sqrt[3]{17}}$

c. $\sqrt[4]{240} = \sqrt[4]{16 \cdot 15} = \underline{2\sqrt[4]{15}}$

d. $\sqrt{75x^3y^8} = \sqrt{25x^2y^8 \cdot 3x}$
 $= \underline{5xy^4\sqrt{3x}}$

e. $\sqrt[3]{135a^7b^{11}} = \sqrt[3]{27a^6b^9 \cdot 5ab^2}$
 $= \underline{3a^2b^3\sqrt[3]{5ab^2}}$

f. $\sqrt[4]{567x^{12}y^{20}} = \sqrt[4]{81x^{12}y^{20} \cdot 7}$
 $= \underline{3x^3y^5\sqrt[4]{7}}$

g. $5\sqrt{7} - 9\sqrt{3} - 6\sqrt{7} + 4\sqrt{3} = \underline{-3\sqrt{7} - 5\sqrt{3}}$

h. $8\sqrt{24} - 4\sqrt{54} + 2\sqrt{150} = 16\sqrt{6} - 12\sqrt{6} + 10\sqrt{6} = \underline{14\sqrt{6}}$

i. $7\sqrt{108} + 5\sqrt{32} - 8\sqrt{500} = 7\sqrt[3]{27 \cdot 4} + 5\sqrt[3]{8 \cdot 4} - 8\sqrt[3]{125 \cdot 4}$
 $= 21\sqrt[3]{4} + 10\sqrt[3]{4} - 40\sqrt[3]{4} = \underline{-9\sqrt[3]{4}}$

j. $\sqrt{\frac{5}{6}} = \frac{\sqrt{5}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{30}}{6}$

k. $5\sqrt{\frac{3}{8}} \cdot 3\sqrt{\frac{2}{9}} = 15\sqrt{\frac{6}{72}} = \frac{15\sqrt{6}}{3\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}}$
 $= \frac{15\sqrt{48}}{3 \cdot 8} = \frac{5\sqrt{48}}{8} = \frac{20\sqrt{3}}{8} = \underline{\frac{5\sqrt{3}}{2}}$

l. $\sqrt[3]{\frac{7}{3}} = \frac{\sqrt[3]{7}}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}} = \frac{\sqrt[3]{63}}{\sqrt[3]{27}}$
 $= \underline{\frac{\sqrt[3]{63}}{3}}$

m. $\sqrt[4]{\frac{2}{9}} = \frac{\sqrt[4]{2}}{\sqrt[4]{9}} \cdot \frac{\sqrt[4]{9}}{\sqrt[4]{9}} = \frac{\sqrt[4]{18}}{\sqrt[4]{81}} = \underline{\frac{\sqrt[4]{18}}{3}}$

Friday
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