6.6 Radical Equations

$\mathbb{R}$

The basic idea behind solving a radical equation is to square, cube, etc each side of the equation to eliminate the radical. As you will see, there are other considerations as well.

$(\sqrt{3x+4})^2 = (5)^2$

$3x+4=25$

$3x=21$

$x=7$ ✓

$D: \{x : x \geq -\frac{4}{3}\}$

even root

$3x+4=0$

$3x=-4$

$x=-4/3$

$(\sqrt[3]{x-5})^3 = (2)^3$

$x-5=8$

$x=13$ ✓

odd roots

$D: \{x : \mathbb{R}\}$

$(\sqrt{x-3})^2 = (x-5)^2$

$x-3=(x-5)(x-5)$

$x-3=x^2-10x+25$

$x^2-11x+28=0$

$(x-7)(x-4)=0$

$x=7, 4$ ext.

Check:

$\sqrt{7-3} \stackrel{?}{=} 7-5$

$\sqrt{4}=2$

$2=2$ ✓

$\sqrt{4-3}=4-5$

$\sqrt{1}=-1$

$1 \neq -1$

NO

Domain:

$D: \{x : x \geq 3\}$

$\sqrt{2x+1}+1=x$

$\quad -1 \quad -1$

$\sqrt{2x+1}=x-1$

$D: \{x : x \geq -\frac{1}{2}\}$

$x = \cancel{0}, 4$

Ext ↑

$2x+1=0$

$2x=-1$

$x=-\frac{1}{2}$

$$\left(\sqrt{2x+3}\right)^2 = \left(\sqrt{x+2}+2\right)^2$$

$$2x+3 = \left(\sqrt{x+2}+2\right)\left(\sqrt{x+2}+2\right)$$

$$2x+3 = (x+2) + 4\sqrt{x+2} + 4$$

$$2x+3 = x+6 + 4\sqrt{x+2}$$

$$(x-3)^2 = \left(4\sqrt{x+2}\right)^2$$

$$x^2-6x+9 = 16(x+2)$$

$$x^2-6x+9 = 16x+32$$

$$x^2-22x-23=0$$

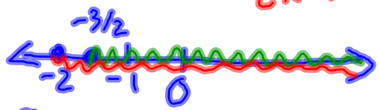
$$(x-23)(x+1)=0$$

$$x = 23, \cancel{-1}$$

Check:

$\cancel{x} = 23$ $\quad x = -1$

↑ Extran.



Domain:

$\sqrt{2x+3}$ $\quad D\{x : x \geq -\frac{3}{2}\}$

$\sqrt{x+2}$ $\quad D\{x : x \geq -2\}$

$$D : \{x : x \geq -\frac{3}{2}\}$$

$$\sqrt{2x+5} + 2\sqrt{x+6} = 5$$

$$\left(\sqrt{x}-\sqrt{7}\right)^2 = \left(\sqrt{x+7}\right)^2$$

$$(\sqrt{x}-\sqrt{7})(\sqrt{x}-\sqrt{7})^2 = x+7$$

$$\cancel{x} - 2\sqrt{7x} + \cancel{7} = \cancel{x} + \cancel{7}$$

$$-2\sqrt{7x} = 0$$

$$\sqrt{7x} = 0$$

$$7x = 0$$

$$x = 0$$

↑ext

$$\sqrt{0}-\sqrt{7} = \sqrt{0+7}$$

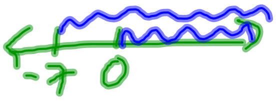
$$-\sqrt{7} = \sqrt{7}$$

X

$\emptyset$

D: $\{x \ge 0\}$ X

D: $\{x \ge -7\}$



$$\left(\sqrt[3]{x^2+x-3}\right)^3 = (x-2)^3$$

$$x^2+x-3 = (x-2)(x-2)(x-2)$$

$$x^2+x-3 = (x-2)(x^2-4x+4)$$

$$x^2+x-3 = x^3-4x^2+4x-2x^2+8x-8$$

$$x^2+x-3 = x^3-6x^2+12x-8$$

$$x^3-7x^2+11x-5=0$$

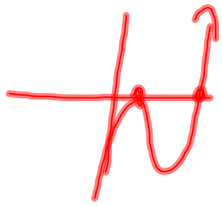
(x _)(x _)(x _)

$$\begin{array}{r|rrrr} 5 & 1 & -7 & 11 & -5 \\ & \downarrow & 5 & -10 & 5 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$$(x-5)(x^2-2x+1)=0$$

$$(x-5)(x-1)(x-1)$$

$x=5$ ✓ $x=1$ ✓



$$\sqrt{x+2}+3=\sqrt{3x-2}$$

When solving an equation with a fractional exponent, isolate the power, and then raise both sides to the reciprocal exponent.

$$\left(x^{\frac{3}{2}}\right)^{2/3} = 8^{2/3}$$

$$x^1 = \left(\sqrt[3]{8}\right)^2$$

$$x = 2^2 = 4$$

$$\frac{5x^{\frac{1}{3}}}{5} = \frac{15}{5}$$

$$\left(x^{\frac{1}{3}}\right)^3 = (3)^3$$

$$x = 27$$

odd roots!

+ -
even roots!!

$(x-2)^{\frac{4}{3}}-5=11$

$\left((x-2)^{4/3}\right)^{3/4}=(16)^{3/4}$

$x-2=\left(\sqrt[4]{16}\right)^{3}$

$x-2=(\pm 2)^{3}$

$x-2=\pm 8$

$x-2=8$ $\quad$ $x-2=-8$

$x=10$ ✓ $\quad$ $x=-6$ ✓

$(2x+3)^{\frac{2}{5}}-1=8$

$(2x+3)^{2/5}=9$

$(2x+3)=9^{5/2}$

$2x+3=(\pm 3)^{5}$

± 243