

ALTERNATING SERIES

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ALTERNATING SERIES

An *Alternating Series* has the form

$$\sum_{n=1}^{\infty} \boxed{} = a_1 - a_2 + a_3 - a_4 + \dots$$

or

$$\sum_{n=1}^{\infty} \boxed{} = -a_1 + a_2 - a_3 + a_4 + \dots$$

note: terms alternate positive..negative or vice-versa
 + - + - + or - + - + - one-by-one
 (all a_n 's are positive)
 Other patterns like + + - - + - - + - -
 are not considered alternating.

EXAMPLES:

(1) Alternating Harmonic Series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{1}{n} \right) = 1 - 1/2 + 1/3 - 1/4 + 1/5 \dots$$

(2)
$$\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n!} \right) = 1 - 1/2! + 1/3! - 1/4! + 1/5! \dots$$

(3)
$$\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{1}{2n^2} \right) = 1/8 - 1/64 + 1/216 - 1/512 \dots$$

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ALTERNATING SERIES TEST (AST) for convergence

Given an Alternating Series like $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$,

if the term sequence (no alternator)

- has limit 0 ($\lim_{n \rightarrow \infty} a_n = 0$)
- and
- is strictly decreasing ($0 < a_{n+1} < a_n$)

then the series **CONVERGES**

Note: The AST tests for convergence, not divergence
 Of course, if $\lim_{n \rightarrow \infty} a_n \neq 0$, then the series diverges by the n^{th} term test

But... if $\lim_{n \rightarrow \infty} a_n = 0$, and the term sequence does not strictly decrease, then you can't really conclude anything... the series could converge or it could diverge

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Nature of convergence for convergent alternating series

Suppose $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ is a convergent alternating series, so its terms decrease strictly to a limit of 0

Look at the sequence of partial sums:

graphically

algebraically

On the graphical representation, indicate the error between S_n and the actual sum S .

Now indicate the distance between the n^{th} sum and the $(n+1)^{\text{st}}$ sum, (which is the same as ?)

Now compare these two quantities.

So

Generalize and put this in words!

Major Observation:

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whose difference is the $n+1^{\text{st}}$ term a_{n+1}

$$|S - s_n| < |s_n - s_{n+1}| = a_{n+1}$$

distance of n^{th} partial sum from sum of series S distance between any two successive partial sums $n+1^{\text{st}}$ term

This also says: if you use s_n (the n^{th} partial sum) to approximate S (the sum of the entire series), your error will always be strictly less than the $n+1^{\text{st}}$ term!

(We use this fact for making approximations to the sum of convergent alternating series, and error-bounding our approximations..i.e. knowing within certain bounds how close our estimate is to the actual sum of the series)

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PROBLEM 1: $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$ 2) $\frac{1}{n+1} < \frac{1}{n} \therefore$ converges by AST. true since $n+1 > n$

Show that the alternating harmonic series converges

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} \quad \text{converges by AST}$$

Note: This series does not converge absolutely

$$\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{1}{n} \right| \quad \text{is really} \quad \sum_{n=1}^{\infty} \frac{1}{n}$$

divergent harmonic series

Q1: In $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ how well does s_{100} approximate the actual sum of the series?

$|S - s_{100}| < a_{101}$
error
 s_{100} is within $\frac{1}{101}$ of true sum of $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{n}$

Q2: In $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ how many terms (n) must you combine to be sure s_n is within 0.001 of the actual sum of the series?

$|S - s_n| < .001$
 $a_{n+1} \leq .001$
 $\frac{1}{n+1} \leq .001$
 $n \geq 999$

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PROBLEM 1:

Show that the alternating harmonic series converges

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} \quad \text{converges by AST}$$

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Problem 2: Show that $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n!}$ converges

$$\frac{1}{1} - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} = \dots$$

then approximate its sum with two decimal place accuracy (i.e. within 5×10^{-3} , or 0.005)

0) Alternating ✓

1) $\lim_{n \rightarrow \infty} \frac{1}{n!} = 0$

$$|S - S_n| < a_{n+1}$$

want $|S - S_n| < .005$

2) $\frac{1}{(n+1)!} < \frac{1}{n!}$

So want $a_{n+1} \leq .005$

since $(n+1)! > n!$

$$\frac{1}{(n+1)!} \leq .005$$

$$\frac{1}{.005} \leq (n+1)!$$

\therefore converges by AST.

$$200 \leq (n+1)!$$

So choose $n=5$

$$6! = 720$$

$$5! = 120$$

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Problem 3: Approximate $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{(2n)^3}$
with 3 decimal place accuracy
(within 5×10^{-4} , or 0.0005)

(when you are finished, compare your result with S_{1000} , which is 0.112692835, as though that is the sum of the series)

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Approximate $\sum_{n=1}^{\infty} (-1)^n \frac{n}{8^n}$ to four decimal places

WATCH THIS!!!!

<http://www.youtube.com/watch?v=iLRkVcAxdCo&feature=channel>



10 min

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