Algebra 2/Trig **Name:** Key

Semester One Exam Review

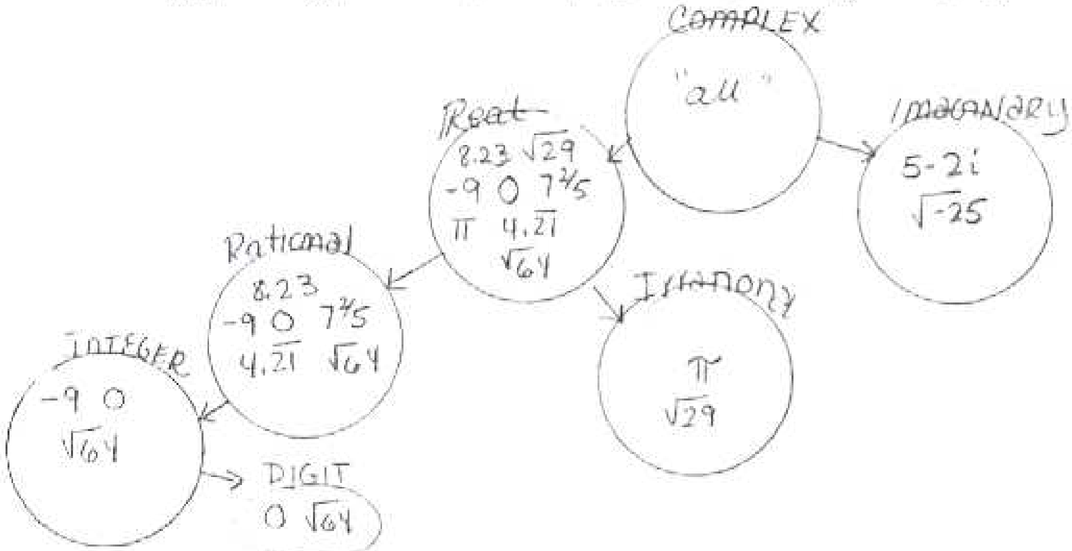
Non-Calculator Part January 2012

1.

Place each number in all sets that it belongs to:

Complex, Real, Imaginary, Rational, Irrational, Integers and Digits

$8.23 \qquad \sqrt{29} \qquad -9 \qquad \sqrt{-25} \qquad 0 \qquad 7\tfrac{4}{5} \qquad 5-2i \qquad \pi \qquad 4.\overline{21} \qquad \sqrt{64}$



2. Simplify each numerical expression. Show all steps and circle your final answer. If the answer is not exact, write it as a mixed number.

a) $24 \div 2^3 \cdot 3 - 6\left[3 - 5(4-7+2)^3\right] = 24 \div 8 \cdot 3 - 6\left[3 - 5(-1)^3\right]$
$= 9 - 6[3+5] = 9 - 6[8] = 9 - 48$
$= -39$

b) $5(6 \div 2 \cdot 3)^2 - 4\left(\dfrac{3^2 - 2^2 + 1^2}{3-2+1}\right)^3 = 5(9)^2 - 4\left(\dfrac{9-4+1}{3-2+1}\right)^3 = 5(81) - 4\left(\dfrac{6}{2}\right)^3$
$= 405 - 4(27) = 405 - 108 = 297$

c) $\left[\dfrac{2+15\div 5}{7-2\cdot 4}\right]^3 - \dfrac{11-2(5-8)^3}{8-12\div 2^2} = \left[\dfrac{2+3}{7-8}\right]^3 - \left(\dfrac{11-2(-3)^3}{8-12\div 4}\right) = \left[\dfrac{5}{-1}\right]^3 - \left(\dfrac{11-2(-27)}{8-3}\right)$
$= -125 - \left(\dfrac{11+54}{5}\right) = -125 - \left(\dfrac{65}{5}\right)$
$= -125 - 13 = -138$

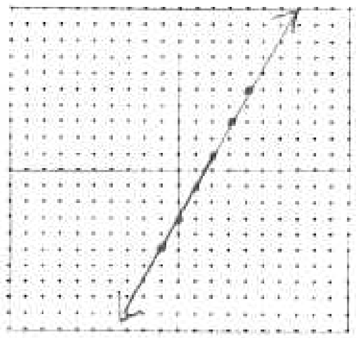
3. Graph each relation by creating a table of values; determine the domain, range, and whether it is a function.

a) $y = 2x - 3$

domain = $\mathbb{R}$ *range* = $\mathbb{R}$

Function? Yes No

x	y
0	-3
1	-1
2	1

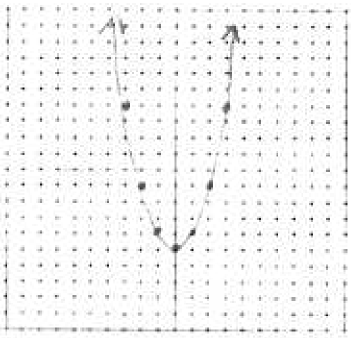


b) $y = x^2 - 5$

domain = $\mathbb{R}$ *range* = $\{y : y \geq -5\}$

Function? Yes No

x	y
0	-5
±1	-4
±2	-1
±3	4



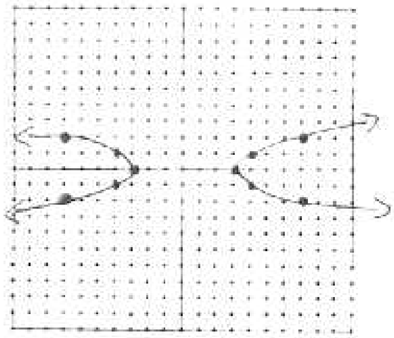
c) $y^2 = |x| - 3$

domain = $\{x : x \geq 3$ and $x \leq -3\}$ *range* = $\mathbb{R}$

Function? Yes No

$y = \pm\sqrt{|x| - 3}$

x	y
3	±0
4	±1
7	±2
-3	±0
-4	±1
-7	±2

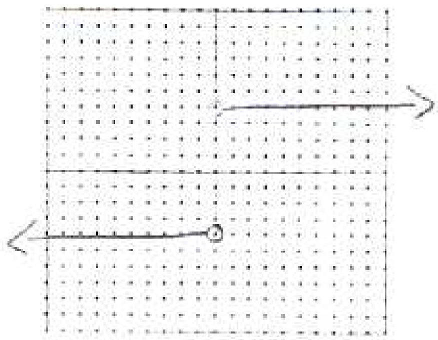


d) $y = \frac{4|x|}{x}$

domain = $\{x : x \neq 0\}$ *range* = $\{y : y = 4, -4\}$

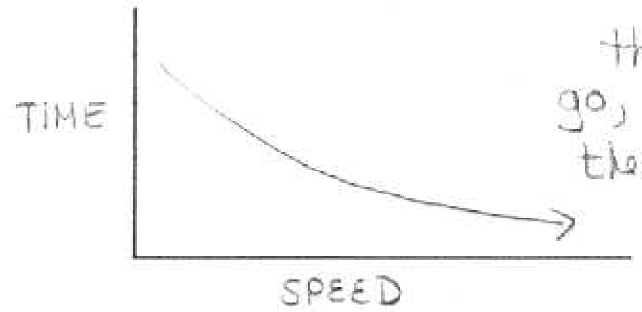
Function? Yes No

x	$y = \frac{4\|x\|}{x}$
-2	-4
-1	-4
0	$\emptyset$
1	4
-2	4



4. Sketch a reasonable graph of the following situations. Decide which value depends on the other value and label the appropriate axis. *Consider what normally happens in the relationship.*

a) The time it takes to get to a destination by car depends on the speed you go.

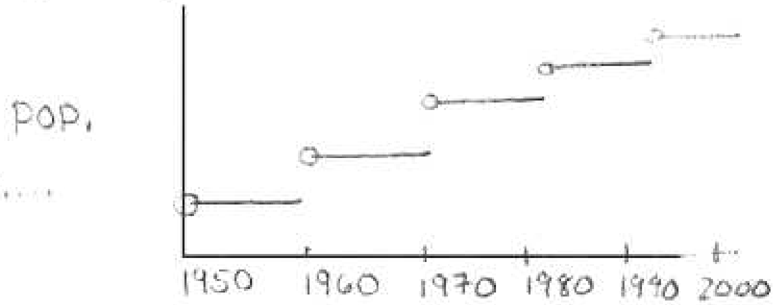


the faster you go, the shorter the time.

b) Your heating bill in Wisconsin depends on the time of year. Assume you begin on January 1.

c) The "posted" population of a town ~~changes~~ increases every 10 years. The posted population depends on the calendar year. Assume this began in the year 1950.

"Posted" population on the welcome to.... sign



5. Find the equation of the line in point-slope form using the given information. Convert the equation to standard form.

a) Contains (-3, 5) with a slope of $-\frac{2}{5}$

$y-5 = -\frac{2}{5}(x+3)$
$5(y-5) = -2(x+3)$
$5y-25 = -2x-6$
$\boxed{2x+5y=19}$

b) Contains (5, -1) and (-1, 3)

$m = \frac{-1-3}{5-(-1)} = \frac{-4}{6} = \frac{-2}{3}$
$y-3 = -\frac{2}{3}(x+1)$
$3y-9 = -2x-2$
$\boxed{2x+3y=7}$

c) Contains (2, 7) and is parallel to $y = -4x + 1$

$m = -4$
$y-7 = -4(x-2)$
$y-7 = -4x+8$
$\boxed{4x+y=15}$

d) Contains (-4, 2) and is perpendicular to $y = \frac{3}{4}x - 5$

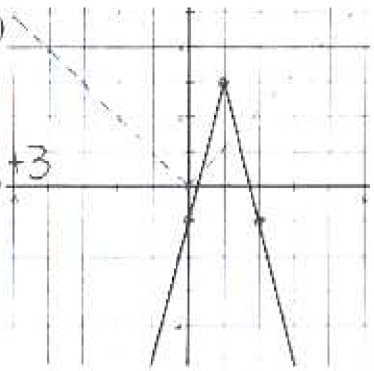
$m = -\frac{4}{3}$
$y-2 = -\frac{4}{3}(x+4)$
$3y-6 = -4x-16$
$\boxed{4x+3y=-10}$

6. Identify the parent function. Write the equation given the graph. [You may check here using your graphing calculator…but not on the test.]

a)

$y=|x|$

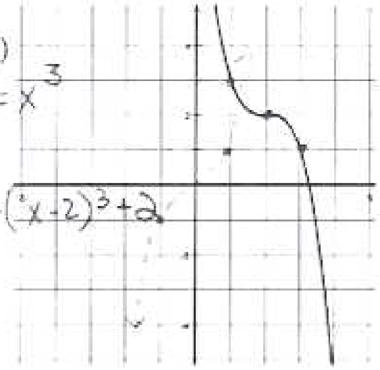
$y=-4|x-1|+3$



b)

$y=x^3$

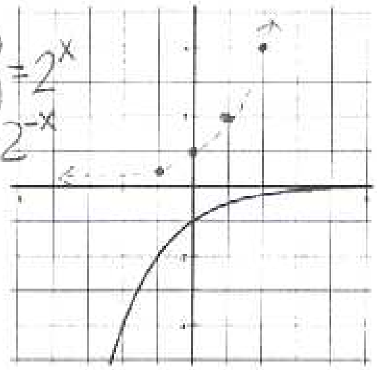
$y=-(x-2)^3+2$



c)

$y=2^x$

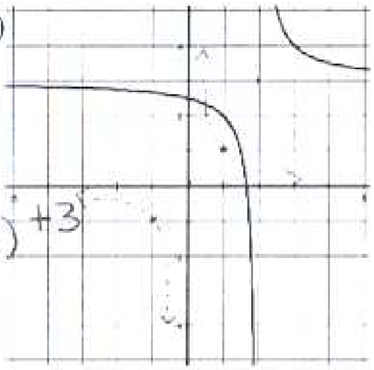
$y=-2^{-x}$



d)

$y=\frac{1}{x}$

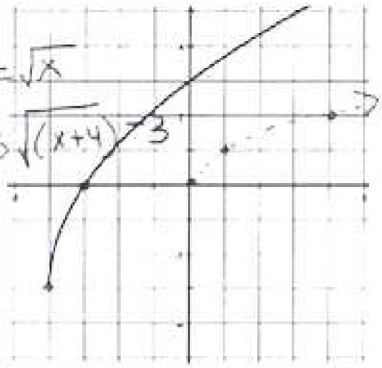
$y=\frac{1}{(x-2)}+3$



e)

$y=\sqrt{x}$

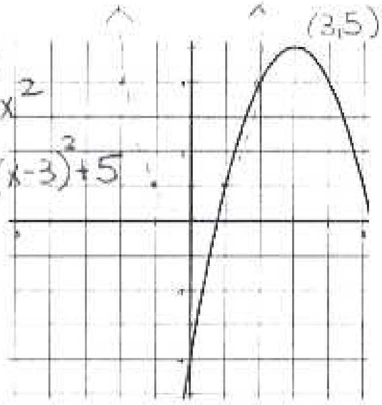
$y=3\sqrt{(x+4)}-3$



f)

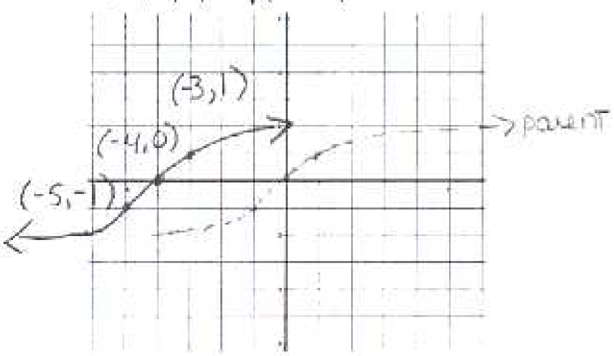
$y=x^2$

$y=-(x-3)^2+5$

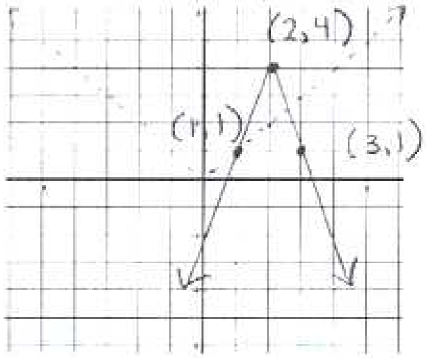


7. Graph the functions below. Label three ordered pairs on each graph.

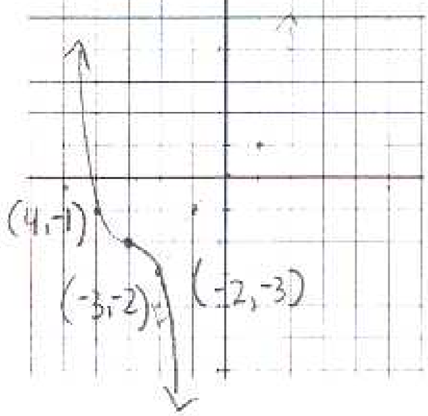
a) $f(x)=\sqrt[3]{(x+4)}$



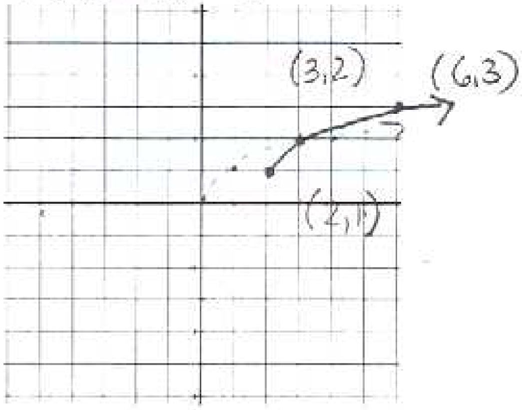
b) $f(x)=-3|x-2|+4$



c) $f(x)=-(x+3)^3-2$



d) $f(x)=\sqrt{(x-2)}+1$



8. For each absolute value function, list the transformations that are made to the parent function $f(x) = |x|$. Transform 3 key points and then check your answer by evaluating these points.

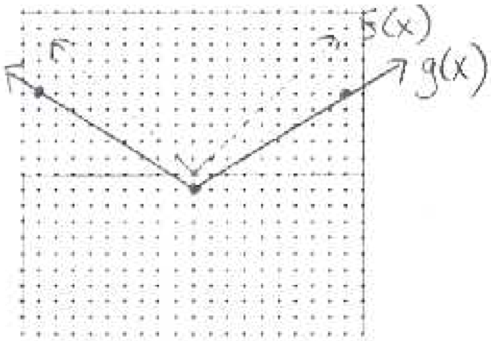
horiz. stretch by 3

a) $g(x) =$ $2\left|\frac{1}{3}x\right| - 1$ down 1

vert stretch by 2

$(0,0) \rightarrow (0,0) \rightarrow (0,0) \rightarrow (0,-1)$
$(3,3) \rightarrow (3,6) \rightarrow (9,6) \rightarrow (9,5)$
$(-3, 3) \rightarrow (-3,6) \rightarrow (-9,6) \rightarrow (-9,5)$

$g(0) = 2|0| - 1 = -1$ ✓
$g(9) = 2|3| - 1 = 5$ ✓
$g(-9) = 2|-3| - 1 = 5$ ✓



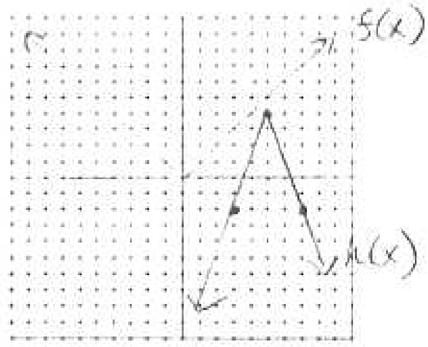
reflect, RIGHT 5

b) $h(x) = -3|x-5| + 4$ UP4

vert. stretch

$(0,0) \rightarrow (0,0) \rightarrow (5,4)$
$(2,2) \rightarrow (2,-6) \rightarrow (7,-2)$
$(-2,2) \rightarrow (-2,-6) \rightarrow (3,-2)$

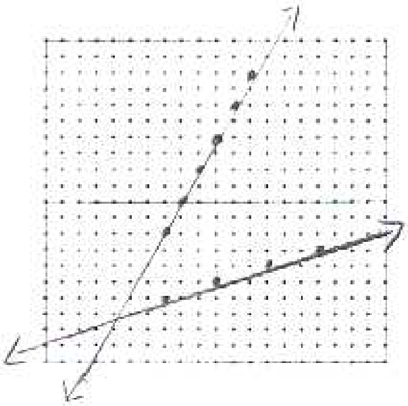
$h(5) = -3|5-5| + 4 = 4$ ✓
$h(7) = -3|7-5| + 4 = -2$ ✓
$h(3) = -3|3-5| + 4 = -2$ ✓



9. Solve the following systems ***graphically*** and check by ***elimination***. Show your work and write all non-integers as mixed numbers.

a) $x - 3y = 15$
$-4x + 2y = 8$

$-3y = -x + 15 \quad y = \frac{1}{3}x - 5$
$2y = 4x + 8 \quad y = 2x + 4$



$4(x - 3y = 15)$
$-4x + 2y = 8$

$\cancel{4x} - 12y = 60$
$\cancel{-4x} + 2y = 8$

$-10y = 68$
$y = -6.8$

$x - 3(-6.8) = 15$
$x + 20.4 = 15$
$x = -5.4$

$(-5.4, -6.8)$

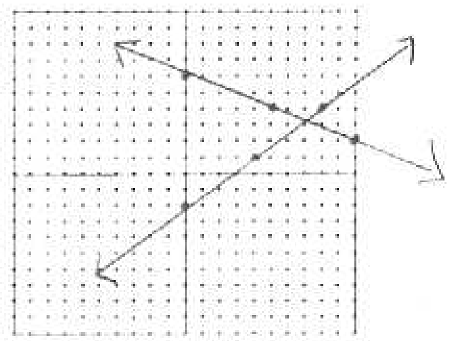
b) $\begin{matrix} 3(& 2x+5y=30 \\ -2(& 3x-4y=8 \end{matrix}$

$$\cancel{6x}+15y=90$$
$$-\cancel{6x}+8y=-16$$
$$23y=74$$
$$\boxed{y=\frac{74}{23}=3\tfrac{5}{23}}$$

can sub back in to find $x=160/23$ or can use elimination to solve for x.

$$8x+2\cancel{0y}=120$$
$$15x-\cancel{20y}=40$$
$$23x=160$$
$$x=160/23$$
$$\boxed{x=6\tfrac{22}{23}}$$

$(6\tfrac{22}{23}, 3\tfrac{5}{23})$



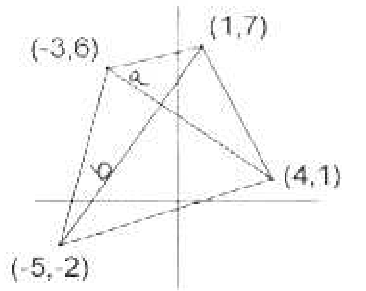
To graph

$2x+5y=30$
$+5y=-2x+30$
$\boxed{y=-\tfrac{2}{5}x+6}$

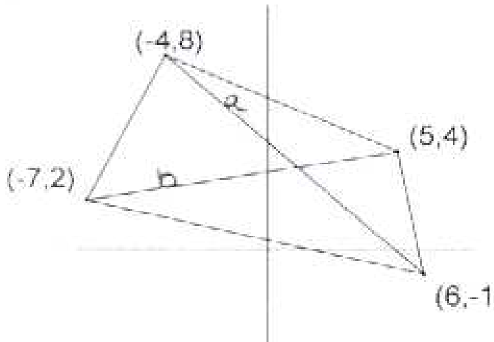
$3x-4y=8$
$-4y=-3x+8$
$\boxed{y=\tfrac{3}{4}x-2}$

10. Find the coordinates of the point where the diagonals of the quadrilateral intersect.

a)



b)



$m_a=\frac{6-1}{-3-4}=\frac{5}{-7}$

$y-1=\frac{-5}{7}(x-4)$
$7(y-1)=-5(x-4)$
$7y-7=-5x+20$
$5x+7y=27$

$m_b=\frac{7-2}{1-5}=\frac{9}{6}=\frac{3}{2}$

$y-7=\tfrac{3}{2}(x-1)$
$2(y-7)=3(x-1)$
$2y-14=3x-3$
$-3x+2y=11$

eliminate twice

$3(5x+7y=27)$
$5(-3x+2y=11)$
$15x+21y=81$
$-15x+10y=55$
$31y=136$
$y=4\tfrac{12}{31}$

$2(5x+7y=27)$
$-7(-3x+2y=11)$
$10x+14y=54$
$21x-14y=-77$
$31x=-23$
$x=-23/31$

$(-\tfrac{23}{31}, 4\tfrac{12}{31})$

$m_a=\frac{8-^-1}{-4-6}=\frac{9}{-10}$

$y+1=\frac{-9}{10}(x-6)$
$10y+10=-9x+54$
$9x+10y=44$

$m_b=\frac{2-4}{-7-5}=\frac{-2}{-12}=\frac{1}{6}$

$y-2=\tfrac{1}{6}(x+7)$
$6y-12=x+7$
$-x+6y=+19$

$9x+10y=44$
$9(-x+6y=19)$
$9x+10y=44$
$-9x+54y=171$
$64y=215$
$y=215/64$
$y=3\tfrac{23}{64}$

$3(9x+10y=44)$
$-5(-x+6y=19)$
$27x+30y=132$
$5x-30y=95$
$32x=37$
$x=37/32$
$x=1\tfrac{5}{32}$

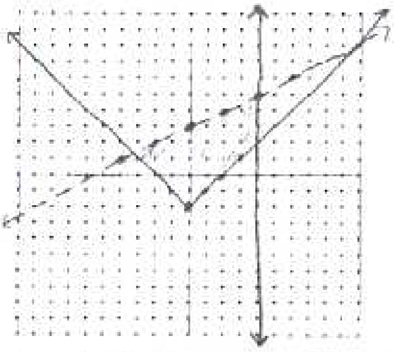
$(1\tfrac{5}{32}, 3\tfrac{23}{64})$

11. Graph the solution set to the following systems of inequalities.

a) $x-2y>-6$ $\quad -2y>-x-6$

$x\le 4$ $\quad y<\frac{1}{2}x+3$

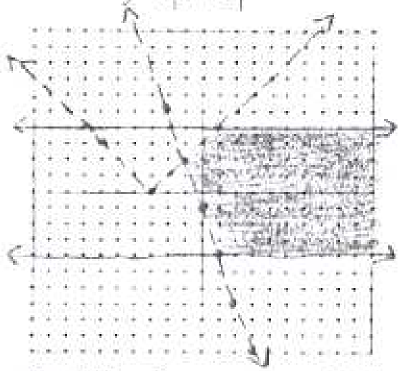
$y\ge |x|-2$



b) $|y|\le 4$

$y>-3x-1$

$y<|x+3|$



12. Use the elimination method to determine the solution to the following systems. Write the solution as an ordered triple.

work to eliminate one variable at a time ☺

a)

$2x-y+3z=1$

$x-4y-2z=-3$

$3x+5y+z=-12$

$x=-3$

$y=-1$ $\quad (-3,-1,2)$

$z=2$

b)

$x+5y+3z=3$

$-2x+4y-z=-2$

$5x-2y+4z=10$

$x=4$

$y=1$ $\quad (4,1,-2)$

$z=-2$

13. Perform the indicated operation. If it cannot be done write "can't be done".

a) $3\begin{bmatrix}-2 & 1 & 5\\ -2 & -3 & 2\end{bmatrix}-2\begin{bmatrix}5 & -4 & 2\\ 3 & 0 & -4\end{bmatrix}=\begin{bmatrix}-6 & 3 & 15\\ -6 & -9 & 6\end{bmatrix}+\begin{bmatrix}-10 & 8 & -4\\ -6 & 0 & 8\end{bmatrix}=\begin{bmatrix}-16 & 11 & 11\\ -12 & -9 & 14\end{bmatrix}$

b) $\underset{3\times 4}{\begin{bmatrix}2 & -1 & 4 & 0\\ -3 & 5 & -2 & 1\\ 4 & 2 & -3 & 4\end{bmatrix}}\cdot\underset{4\times 2}{\begin{bmatrix}-3 & 1\\ 2 & 5\\ -4 & 0\\ 2 & -1\end{bmatrix}}=\begin{bmatrix}-6-2-16+0 & 2-5+0+0\\ 9+10+8+2 & -3+25+0-1\\ -12+4+12+8 & 4+10+0-4\end{bmatrix}=\begin{bmatrix}-24 & -3\\ 29 & 21\\ 12 & 10\end{bmatrix}$

c) $\begin{bmatrix}5 & -1 & 2\\ 4 & -3 & 0\end{bmatrix}\cdot\begin{bmatrix}1 & -3\\ 2 & -4\\ 5 & -1\end{bmatrix}+5\begin{bmatrix}3 & -4\\ 2 & -1\end{bmatrix}=\begin{bmatrix}5-2+10 & -15+4-2\\ 4-6+0 & -12+12+0\end{bmatrix}+\begin{bmatrix}15 & -20\\ 10 & -5\end{bmatrix}$

$=\begin{bmatrix}13 & -13\\ -2 & 0\end{bmatrix}+\begin{bmatrix}15 & -20\\ 10 & -5\end{bmatrix}=\begin{bmatrix}28 & -33\\ 8 & -5\end{bmatrix}$

(KEY)

14. Given the equation in vertex form, determine the vertex. Make a table of values to graph the parabola, using symmetry to get the other side. Show the substitution of the values into the equation.

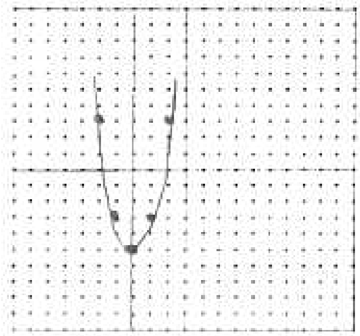
a) $y = 2(x+3)^2 - 5$

$= 2\left(x-(-3)\right)^2 + (-5)$

VERTEX $(-3,-5)$

x	y
−4	−3
−5	3
−2	−3
−1	3

BY SYMMETRY

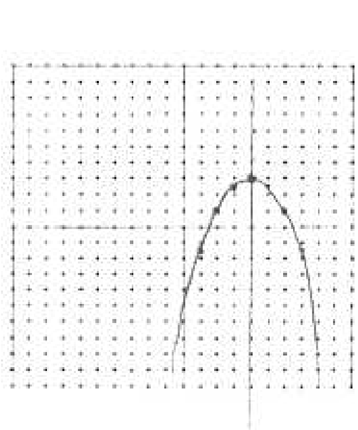


b) $y = -\frac{1}{2}(x-4)^2 + 3$

VERTEX: $(4,3)$

x	y
3	2.5
2	1
1	−1.5
5	2.5
6	1

BY SYMM



$-b/2a$

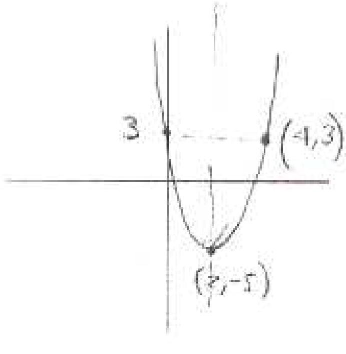
15. Use the formula for the vertex to determine the vertex. Find the y-intercept and a point symmetric to the y-intercept. Sketch the parabola by plotting and labeling these points.

X COORDINATE OF VERTEX: $\frac{-b}{2a}$

a) $y = 2x^2 - 8x + 3$

X_v: $\frac{8}{2(2)} = \frac{8}{4} = 2$

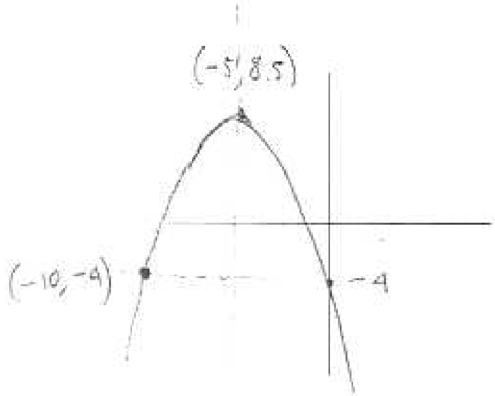
Y_v: -5



b) $y = -\frac{1}{2}x^2 - 5x - 4$

X_v: $\frac{5}{-1} = -5$

Y_v: 8.5



16. Perform the indicated operation.

a) $(7-3i)+(-5+4i) = 2+i$

b) $(-1+7i)-(8+11i) = -9-4i$

c) $(3+4i)(2-5i) = 6-7i-20i^2 = 26-7i$

d) $\frac{7+i}{3-2i} \cdot \frac{3+2i}{3+2i} = \frac{21+17i+2i^2}{13} = \frac{19}{13}+\frac{17}{13}i$

17. Evaluate each of the following.

$i^0=1$ $i^4=1$
$i^1=i$
$i^2=-1$
$i^3=-i$

a) $i^{41} = i$

b) $i^{22} = -1$

18. Solve each equation below using the quadratic formula. Write all complex roots in the form $a+bi$. Show a check of one of the solutions.

a) $x^2-2x+17=0$

$x = \frac{2 \pm \sqrt{4-4\cdot1\cdot17}}{2} = \frac{2\pm\sqrt{-64}}{2}$

$= 1 \pm 4i$

b) $x^2+6x+34=0$

$x = \frac{-6 \pm \sqrt{36-4\cdot1\cdot34}}{2}$

$= \frac{-6 \pm \sqrt{-100}}{2}$

$= -3 \pm 5i$

19. Evaluate each of the following.

a) $(-2)^{-3} = \frac{1}{(-2)^3} = -\frac{1}{8}$

b) $\left(\frac{3}{7}\right)^{-2} = \frac{7^2}{3^2} = \frac{49}{9}$

c) $5^0+(-4)^0 = 2$

d) $\left(\frac{3}{4}\right)^{-2} - \left(\frac{6}{5}\right)^{-1} = \frac{4^2}{3^2} - \frac{5}{6}$

$= \frac{16}{9} - \frac{5}{6} = \frac{32-15}{18}$

$= \frac{17}{18}$

e) $\left(5^{-2}+10^{-1}\right)^{-2} = \left(\frac{1}{5^2}+\frac{1}{10}\right)^{-2} = \left(\frac{7}{50}\right)^{-2} = \frac{50^2}{7^2}$

20. Simplify each expression. Write your answer without negative exponents.

a) $(7x^4y^2)(-6x^{-5}y^3)$

$= -42x^{-1}y^5 = \frac{-42y^5}{x}$

b) $\frac{18a^4b^{-3}}{-6a^{-1}b^{-5}} = -3a^5b^2$

c) $(4x^2y^{-5})^2(-3x^{-4}y)^{-3}$

$\frac{4^2x^4y^{-10}}{(-3x^{-4}y)^3} = \frac{16x^4y^{-10}}{(-3)^3x^{-12}y^3} = -\frac{16}{27}\frac{x^{16}}{y^{13}}$

d) $\frac{-20a^7b^2c^{-6}}{30a^{-4}b^8c^3} = -\frac{2}{3}\frac{a^{11}}{b^6c^9}$

e) $\frac{(5x^6y^{-1})^{-2}}{(10x^{-4}y^7)^{-1}}$

$= \frac{10x^{-4}y^7}{(5x^6y^{-1})^2} = \frac{10x^{-4}y^7}{5^2x^{12}y^{-2}} = \frac{2y^9}{5x^{16}}$

f) $\left(\frac{12x^{-3}y}{3x^2y^{-4}}\right)^{-3} = \left(4x^{-5}y^5\right)^{-3}$

$= 4^{-3}x^{15}y^{-15}$

$= \frac{x^{15}}{4^3y^{15}}$

2. Perform the indicated operation.

a) $(3x-2)(5x+2)(x-4) = (15x^2-4x-4)(x-4) \Rightarrow 15x^3-4x^2-4x-60x^2+16x+16$

$\Rightarrow 15x^3-64x^2+12x+16$

b) $(2x^2-5x+3)(x^2+4x-1)$

$\Rightarrow 2x^4+8x^3-2x^2-5x^3-20x^2+5x+3x^2+12x-3 \Rightarrow 2x^4+3x^3-19x^2+17x-3$

c) $(7x-3)^2 = 49x^2-42x+9$

d) $(2x-5)^3 = 8x^3-60x^2+150x-125$

2. Factor each of the following using a combination of factoring methods. Show each step and write the final factors on the line.

GCF trinomial

a) $21n^3+35n^2-84n$

$= 7n(3n^2+5n-12) = 7n(3n-4)(n+3)$ ____________

trinomial diff sq. diff sq. diff sq.

b) x^4-50x^2+49 $\underline{(x+1)(x-1)(x+7)(x-7)}$

$(x^2-1)(x^2-49)$

$(x+1)(x-1)(x+7)(x-7)$

group diff. sq.

c) $4a^3+8a^2-9a-18$ $\underline{(2a+3)(2a-3)(a+2)}$

$\Rightarrow 4a^2(a+2)-9(a+2) \Rightarrow (4a^2-9)(a+2)$

$(2a+3)(2a-3)(a+2)$

GCF diff cubes

d) $24a^4-375a = 3a(8a^3-125)$ $\rightarrow$ $\underline{3a(2a-5)(4a^2+10a+25)}$

$= 3a((2a)^3-(5)^3)$

diff sq. diff sq.

e) $81a^4-16 = (9a^2-4)(9a^2+4)$ $\rightarrow$ $\underline{(3a+2)(3a-2)(9a^2+4)}$

$(3a+2)(3a-2)(9a^2+4)$

trinomial subst.

f) $a^2-14a+49-b^2$ $x^2-b^2=(x+b)(x-b)$ $\underline{((a-7+b)(a-7-b)}$

sub back in

$\Rightarrow (a-7)^2-b^2$

let $x=a-7$

g) $2(a+b)^2-9(a+b)+10$ LET $(a+b)=A$ $\underline{(2a+2b-5)(a+b-2)}$

subst.

$2A^2-9A+10$

$= (2A-5)(A-2)$

(23) Find a polynomial function with a positive leading coefficient with zeros of $-\frac{1}{3}$, $\frac{2}{5}$, and 4. Rely on the end behavior, the x-intercepts, and the y-intercepts to sketch the graph of the function.

$f(x) = (x+\frac{1}{3})(x-\frac{2}{5})(x-4)$

FACTOR THEOREM $\Rightarrow (3x+1)(5x-2)(x-4)$

$= (15x^2-x-2)(x-4)$

$= 15x^3-x^2-2x-60x^2+4x+8$

$= 15x^3-61x^2+2x+8$

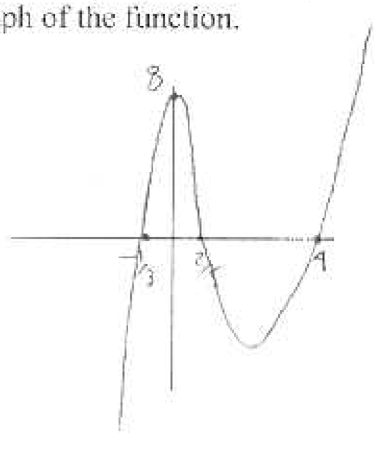
REMEMBER: YOU ARE SKETCHING WITHOUT A CALCULATOR!

3RD DEGREE POLYNOMIAL WITH POSITIVE LEADING COEFF.

GENERAL SHAPE

X-INTERCEPTS: SET $y=0$ → "ZEROS"

Y INTERCEPT: SET $x=0$, $y=8$



(24) Find a polynomial function with a negative leading coefficient with zeros of $\frac{3}{4}$, $-\frac{1}{2}$, and a double root of 3. Rely on the end behavior, the x-intercepts, and the y-intercepts to sketch the graph of the function.

$f(x) = (x-\frac{3}{4})(x+\frac{1}{2})(x-3)^2$

FACTOR TH $\Rightarrow (4x-3)(2x+1)(x-3)^2$

FOR A NEGATIVE LEADING COEFF

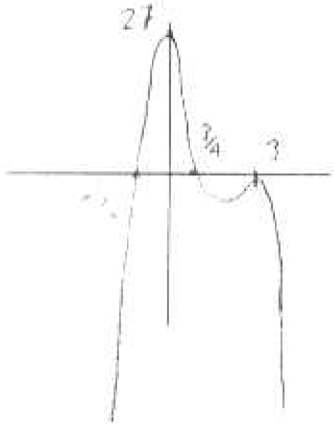
4 TH DEGREE POLY. WITH NEG LC

GENERAL SHAPE

"ZERO'S" AT $y=0$

DOUBLE ROOT IMPLIES THAT FUNCTION TOUCHES THE X-AXIS ONLY

Y INT: -27



$f(x) = -1(4x-3)(2x+1)(x-3)^2$

$= -1(8x^2-2x-3)(x^2-6x+9)$

$= -1(8x^4-48x^3+72x^2-2x^3+12x^2-18x-3x^2+18x-27)$

$= -1(8x^4-50x^3+81x^2-27) = -8x^4+50x^3-81x^2+27$

(25) Evaluate each of the following. Show the key step and circle your final answer.

a) $8^{\frac{2}{3}} = (2^3)^{2/3} = 2^2 = 4$

b) $16^{-\frac{3}{4}} = (2^4)^{-3/4} = 2^{-3} = \frac{1}{8}$

OR $(\sqrt[4]{16})^{-3} = (2)^{-3} = \frac{1}{8}$

c) $(-25)^{\frac{3}{2}} = (-1)^{3/2} \cdot (5^2)^{3/2}$

NOT POSSIBLE IN THE REAL NUMBER SET

$-125i$

d) $(-32)^{-\frac{3}{5}} \Rightarrow (\sqrt[5]{-32})^{-3} = (-2)^{-3} = \frac{1}{(-2)^3}$

$= -\frac{1}{8}$

e) $\left(\frac{125}{27}\right)^{\frac{4}{3}}$

$= \left(\frac{5^3}{3^3}\right)^{4/3} = \frac{5^4}{3^4} = \frac{625}{81}$

f) $\left(-\frac{1}{8}\right)^{-\frac{1}{3}}$

$\frac{1}{(-\frac{1}{8})^{1/3}} = \frac{1}{\sqrt[3]{-\frac{1}{8}}} = \frac{1}{-\frac{1}{2}} = -2$

Calculator Part

26. Solve the following absolute value inequality algebraically. You do not need to use the interval method. ~~Show a graphing calculator solution on the graph provided and a~~ final solution set on the number line. Show all steps and be sure to label the solutions and functions graphed.

a) $5|x-4|+2 \geq 17$

$\Rightarrow 5|x-4| \geq 15$

$\Rightarrow |x-4| \geq 3$

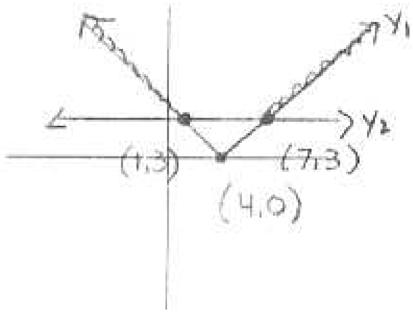
$Y_1 = |x-4|$

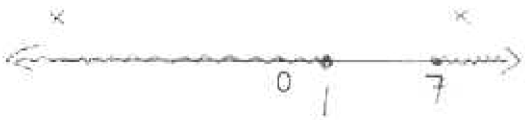
$y_2 = 3$

THE QUANTITY $(x-4)$ HAS TO BE ≥ 3 OR ≤ -3 IN ORDER FOR $(x-4)$ TO BE AT A DISTANCE OF 3 OR GREATER FROM 0

$x-4 \geq 3$ OR $x-4 \leq -3$

$x \geq 7$ OR $x \leq 1$





b) $7-2|3x+1| < -5$

$-2|3x+1| < -12$

$|3x+1| > 6$

$Y_1 = |3x+1|$

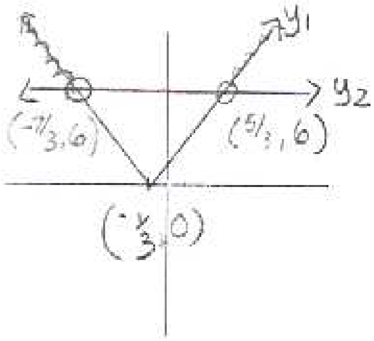
$y_2 = 6$

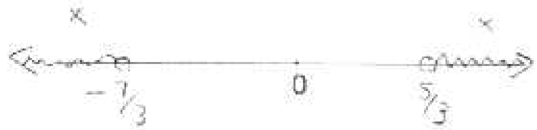
THE QUANTITY $(3x+1)$ HAS TO BE >6 OR $(3x+1)$ HAS TO BE <-6. IN ORDER FOR $(3x+1)$ TO BE AT A DISTANCE OF 6 OR GREATER FROM 0.

$3x+1 > 6$ OR $3x+1 < -6$

$3x > 5$ $3x < -7$

$x > 5/3$ $x < -7/3$





27. Solve the following inequality using the ***interval method.*** Show a graphing calculator solution on the graph provided and a final solution on the number line. Show all steps and be sure to label the solutions and functions graphed.

a) $\underset{①}{|2x+1|} - \underset{②}{|x-2|} \ge 3$

①	−	+	+
②	−	−	+
	$-\frac{1}{2}$		2

If $x \le -\frac{1}{2}$

$-(2x+1) - ^-(x-2) \ge 3$

$-2x+1+x-2 \ge 3$

$-x-3 \ge 3$

$-x \ge 6$

then $x \le -6$

-6 0

If $-\frac{1}{2} < x \le 2$

$(2x+1) - ^-(x-2) \ge 3$

$2x+1+x-2 \ge 3$

$3x-1 \ge 3$

$3x \ge 4$

then $x \ge \frac{4}{3}$

4/3 2

If $x > 2$

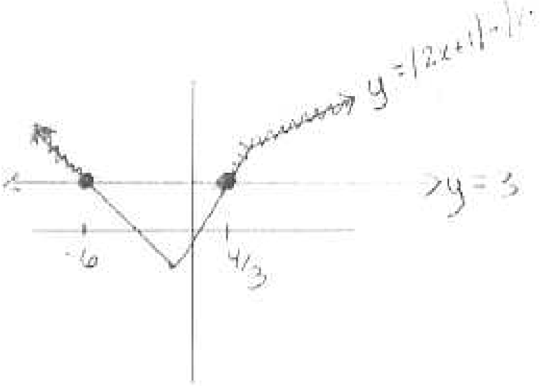
$(2x+1)-(x-2) \ge 3$

$2x+1-x+2 \ge 3$

$x+3 \ge 3$

then $x \ge 0$

2



-6 0 4/3

b) $\underset{①}{|x-3|} + \underset{②}{|3x-4|} < \underset{③}{|x+2|}$

①	−	−	−	+
②	−	−	+	+
③	−	+	+	+
	−2	4/3	3	

If $x \le -2$

$-(x-3) - (3x-4) < -(x+2)$

$-x+3-3x+4 < -x-2$

$-4x+7 < -x-2$

$-3x < -9$

then $x > 3$

$\emptyset$

-2 0 3

If $-2 < x \le \frac{4}{3}$

$-(x-3)-(3x-4) < (x+2)$

$-x+3-3x+4 < x+2$

$-4x+7 < x+2$

$-5x < -5$

$x > 1$

-2 0 1 4/3

If $\frac{4}{3} < x \le 3$

$-(x-3)+(3x-4) < (x+2)$

$-x+3+3x-4 < x+2$

$2x-1 < x+2$

then $x < 3$

4/3 3

If $x > 3$

$(x-3)+(3x-4) < (x+2)$

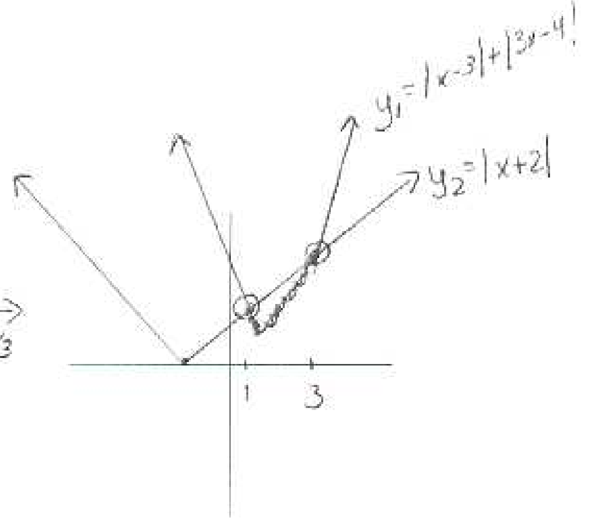
$x-3+3x-4 < x+2$

$4x-7 < x+2$

$3x < 9$

then $x < 3$

$\emptyset$



0 1 4/3 3

26. Car Rental Problem: You rent a car and take a plan where you pay by the mile. If you drive 40 miles you pay \$13 and you pay \$21.80 for driving 80 miles. Assume that the cost varies linearly with the miles driven.

a) What is the slope and what does it represent in the real world?

$$\text{Slope} = \frac{21.80 - 13}{80 - 40} = .22 \qquad \frac{\$.22}{\text{mile}}$$

b) Find the equation that relates the cost to the number of miles driven.

$$y - 13 = .22(x - 40)$$

$$y = .22x + 4.20$$

c) Find the cost for driving 200 miles.

\$48.20

d) How far would you drive to have a cost of \$100?

$435.\overline{45}$ miles

e) What is the cost-intercept and what does it represent in the real world?

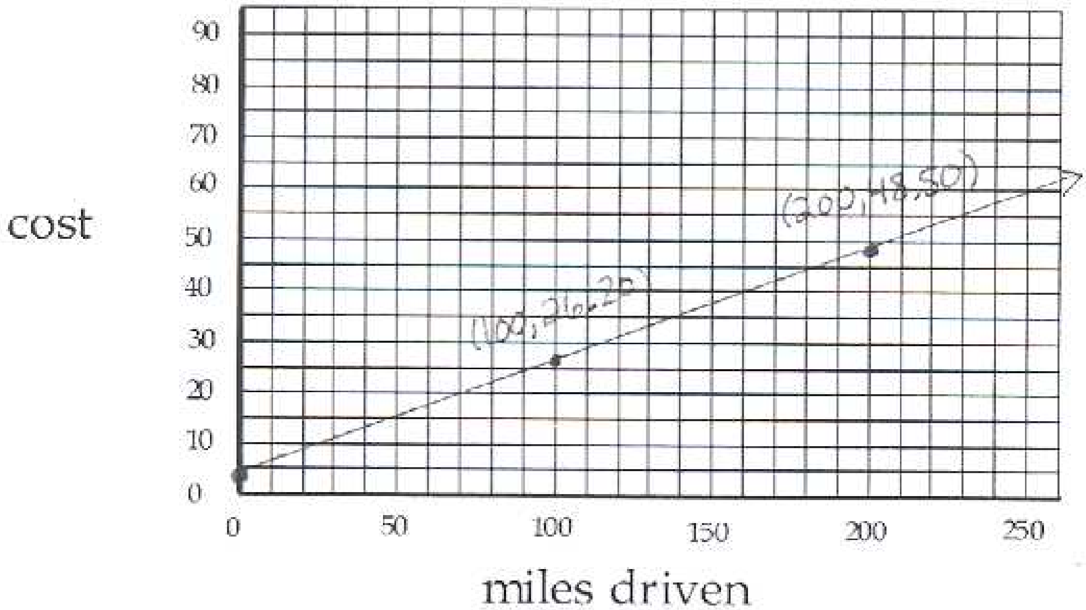
\$4.20 is the cost for 0 miles.

\$4.20 is the flat fee.

f) What is the miles-intercept and what does it represent in the real world?

-19.09 meaningless

g) Sketch a graph of the function. Note that the graph has already been labeled



29. Write each matrix multiplication problem as a system of equations.

a) $\begin{bmatrix} 2 & -4 \\ 5 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$

$2x - 4y = 7$
$5x - y = -4$

b) $\begin{bmatrix} 3 & -2 & -4 \\ 2 & 1 & 5 \\ -4 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -3 \\ 2 \end{bmatrix}$

$3x - 2y - 4z = 10$
$2x + 1y + 5z = -3$
$-4x + 2y - 1z = 2$

30. Solve each system by using matrices. Show how the problem is set up. All non-integer answers should be written as fractions.

a)

$3x - y + 2z = 1$
$x - 4y - 5z = 6$
$2x + 5y + z = -3$

$$\begin{bmatrix} 3 & -1 & 2 \\ 1 & -4 & -5 \\ 2 & 5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ -3 \end{bmatrix}$$

$A \cdot X = B$

$X = A^{-1} \cdot B$

$$X = \begin{bmatrix} 12/25 \\ -17/25 \\ -14/25 \end{bmatrix}$$

b)

$2a - b + 3c + 2d = 2$
$a + 5b - 4d = 1$
$3a - 4b + c + 2d = -5$
$5a - 3c + d = 4$

$$\begin{bmatrix} 2 & -1 & 3 & 2 \\ 1 & 5 & 0 & -4 \\ 3 & -4 & 1 & 2 \\ 5 & 0 & -3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -7/137 \\ 356/137 \\ -58/137 \\ 409/137 \end{bmatrix}$$

31. During a basketball season, Devin Harris made a total of 880 points. The total number of shots made was 420, which was a combination of 3-pointers, 2-pointers, and free throws. The number of 2-point shots made was the sum of the number of 3-pointers and free throws. Find the number of shots of each type that he made. x = # of 3-pt shots, y = # of 2-pt shots, z = # of free throws

$3x + 2y + z = 880$
$x + y + z = 420$
$y = x + z$
$-x + y - z = 0$

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 880 \\ 420 \\ 0 \end{bmatrix}$$

125 3pt shots, 210 2pt shots, 85 free throws

32. At the school performance of Grease, student tickets sold for \$2 each, adults were \$5 each, and senior citizens were \$3 each. There were 150 sold for one performance and the total revenue was \$600. The number of adult tickets equaled the number of student tickets plus the number of senior citizen tickets. Find the number of each ticket sold for this performance.

x = # student tickets, y = # adult tickets, z = # senior citizen tickets

$2x + 5y + 3z = 600$
$x + y + z = 150$
$y = x + z$
$-x + y - z = 0$

0 student tickets
75 adult tickets
75 senior citizen tickets

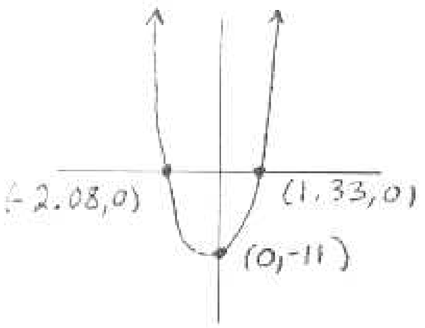
33. Use your graphing calculator to sketch the graph of each quadratic function. Use the quadratic formula to determine the x-intercepts of the graph. Write each value to 2 decimal places.

a) $y = 4x^2 + 3x - 11$

$$x = \frac{-3 \pm \sqrt{9 - 4(4)(-11)}}{2(4)}$$

$$x = \frac{-3 \pm \sqrt{185}}{8}$$

$x \approx 1.33$, $x \approx -2.08$

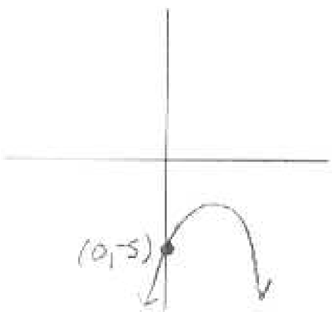


b) $y = -x^2 + 2x - 5$

$$x = \frac{-2 \pm \sqrt{4 - 4(-1)(-5)}}{2(-1)}$$

$$x = \frac{-2 \pm \sqrt{-16}}{-2}$$

$x = \emptyset$ in $\mathbb{R}$, no x intercepts



34. Determine a quadratic function that contains the given points. Write all non-integer values as fractions.

$y = ax^2 + bx + c$

a) (-2, 12.4) (3, -3.6) (6, 6)

$4a + {-2b} + c = 12.4$
$9a + 3b + c = -3.6$
$36a + 6b + c = 6$

$y = \frac{4}{5}x^2 - 4x + \frac{6}{5}$

b) (-6, 1) (-3, 7.5) (5, -4.5)

$36a - 6b + c = 1$
$9a - 3b + c = 7.5$
$25a + 5b + c = -4.5$

$y = -\frac{1}{3}x^2 - \frac{5}{6}x + 8$

35. Fireworks are shot off from a platform 3 feet high with an initial velocity of 144 feet per second. The height of the fireworks as a function of time is given by the formula below. Will the fireworks reach a height of 330 feet above the ground? Determine the answer in *two different ways.*

$h = -16t^2 + 144t + 3$

Max:

$$t = \frac{-144}{2(-16)} = 4.5$$

$h = -16(4.5)^2 + 144(4.5) + 3$

$h = 327$ feet

no

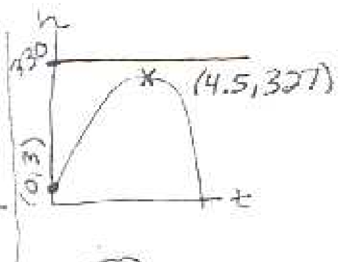
$330 = -16t^2 + 144t + 3$

$0 = -16t^2 + 144t - 327$

$$t = \frac{-144 \pm \sqrt{144^2 - 4(-16)(-327)}}{2(-16)}$$

$$t = \frac{-144 \pm \sqrt{-192}}{-32} = \emptyset \text{ in } \mathbb{R}$$

no



no

36. A rectangular corral is built for a horse that uses a barn as one side and 150 feet of fencing for the other three sides. Let x represent one of the equal sides of the corral.

a) What is the expression for the length of the other side of the corral?

$\boxed{150-2x}$

b) Write a quadratic function for the area $A(x)$ of the coral. Convert this to standard form.

$A(x) = x(150-2x)$ → $\boxed{A(x) = -2x^2 + 150x}$

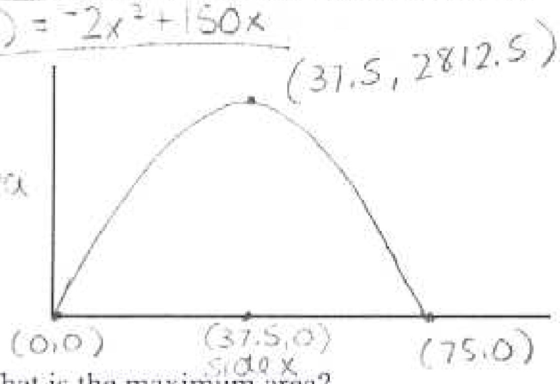
$= 150x - 2x^2$

* factored form - intercepts at (0,0) (75,0)

c) Sketch a graph of this relationship labeling the axes and the window.

vertex @ $x = 37.5$

$A(x) = 2812.50$



d) What dimensions give the maximum area? What is the maximum area?

$x = 37.5$ ft.

$150 - 2x = 75$ ft. $\boxed{37.5 \text{ by } 75 \text{ ft.}}$ $\boxed{A(x) = 2812.50 \text{ sq. ft.}}$

37. The gas mileage for cars is low for slow speeds, increases for moderate speeds, and decreases again at high speeds. The gas mileage for speeds of 20 mph, 40 mph, and 70 mph are 15 mpg, 24mpg, and 21 mpg respectively.

gas mileage depends on speed (20, 15) (40, 24) (70, 21)

given 3 points write 3 equations

a) Write a quadratic model for this situation. Write the coefficients as decimals(3 decimal places).

$y = ax^2 + bx + c$

$15 = a(20)^2 + b(20) + c$

$24 = a(40)^2 + b(40) + c$

$21 = a(70)^2 + b(70) + c$

matrices $\begin{bmatrix} 400 & 20 & 1 \\ 1600 & 40 & 1 \\ 4900 & 70 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 24 \\ 21 \end{bmatrix}$ $X = A^{-1} \cdot B$

$a = -.011$

$b = 1.11$

$c = -2.8$

b) What is the gas mileage for a speed of 50 miles per hour?

$\boxed{y = -.011x^2 + 1.11x - 2.8}$

OR could enter data in L1 L2 & do quad regression

$x = 50$ mph (max)

$y = 25.2$

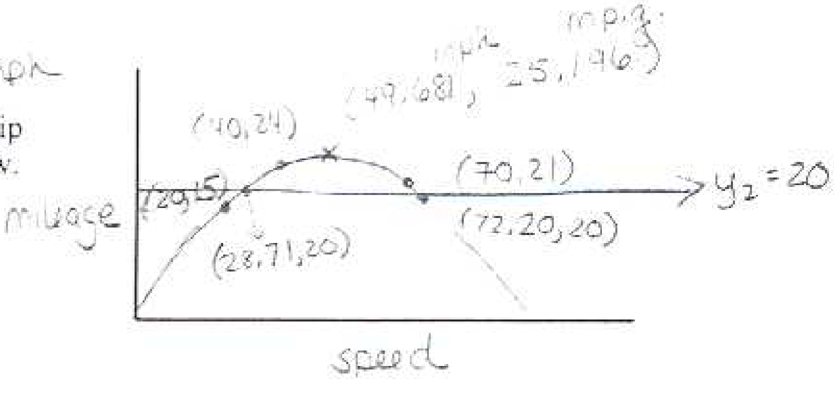
c) For what speeds would the mileage be at least 20 miles per gallon?

$y_1 = -.011x^2 + 1.11x - 2.8$

$y_2 = 20$

$28.71 \le x \le 72.20$ mph

d) Sketch a graph of this relationship labeling the axes and the window.



e) At what speed do you get the best gas mileage? What is this mileage?

vertex (49.681, 25.196)

(50, 25.2)
mph mpg

38. Consider the following polynomials. Show the given numbers are rational zeros using synthetic division. Solve the resulting quadratic to find all other rational, irrational, or imaginary solutions. Write all irrational solutions in simplified radical form and all imaginary values in the form $a \pm bi$.

Be able to sketch any polynomial with rational or irrational solutions using end behavior, zeros and y-intercept.

a) $f(x) = x^3 - 5x^2 - 8x + 12$ with a rational root of -2.

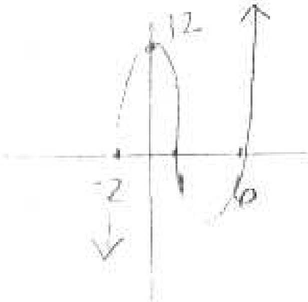
rational roots

$$\begin{array}{r|rrrr} -2 & 1 & -5 & -8 & 12 \\ & \downarrow & -2 & 14 & -12 \\ \hline & 1 & -7 & 6 & 0 \end{array}$$

$(x+2)(x^2-7x+6)=0$

$(x+2)(x-6)(x-1)=0$

$x = -2, 6, 1$



b) $f(x) = 8x^3 + 22x^2 - 25x + 3$ with a rational root of $\frac{3}{4}$

one rational 2 irrational roots

$$\begin{array}{r|rrrr} 3/4 & 8 & 22 & -25 & 3 \\ & \downarrow & 6 & 21 & -3 \\ \hline & 8 & 28 & -4 & 0 \end{array}$$

$(x-3/4)(8x^2+28x-4)=0$

$(x-3/4)(4)(2x^2+7x-1)=0$

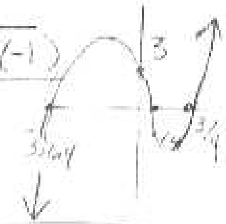
$(4x-3)(2x^2+7x-1)=0$

$x = \dfrac{-7 \pm \sqrt{49-4(2)(-1)}}{2(2)}$

$x = \dfrac{-7 \pm \sqrt{57}}{4}$

$x = 3/4, \dfrac{-7+\sqrt{57}}{4}, \dfrac{-7-\sqrt{57}}{4}$

$\approx .14$ ≈ -3.64



c) $f(x) = 2x^4 - 21x^3 + 71x^2 - 61x - 51$ with rational roots of 3 and $-\frac{1}{2}$

two rational two imaginary

$$\begin{array}{r|rrrrr} 3 & 2 & -21 & 71 & -61 & -51 \\ & \downarrow & 6 & -45 & 78 & 51 \\ \hline -1/2 & 2 & -15 & 26 & 17 & 0 \\ & \downarrow & -1 & 8 & -17 & \\ \hline & 2 & -16 & 34 & 0 & \end{array}$$

$(x-3)(x+\frac{1}{2})(2x^2-16x+34)=0$

$(x-3)(x+\frac{1}{2})(2)(x^2-8x+17)=0$

$(x-3)(2x+1)(x^2-8x+17)=0$

$x = \dfrac{8 \pm \sqrt{(-8)^2-4(1)(17)}}{2(1)}$

$x = \dfrac{8 \pm \sqrt{64-68}}{2}$

$x = \dfrac{8 \pm \sqrt{-4}}{2} = \dfrac{8 \pm 2i}{2} = 4 \pm i$

$x = 3, -\frac{1}{2}, 4+i, 4-i$

d) $f(x) = 15x^4 + 73x^3 - 91x^2 - 41x - 4$ with rational roots of $\frac{4}{3}$ and $-\frac{1}{5}$

two rational two irrational

$$\begin{array}{r|rrrrr} 4/3 & 15 & 73 & -91 & -41 & -4 \\ & \downarrow & 20 & 124 & 44 & 4 \\ \hline & 15 & 93 & 33 & +3 & 0 \\ -1/5 & \downarrow & -3 & -18 & -3 & \\ \hline & 15 & 90 & 15 & 0 & \end{array}$$

$(x-4/3)(x+\frac{1}{5})(15x^2+90x+15)=0$

$(x-4/3)(x+\frac{1}{5})(15)(x^2+6x+1)=0$

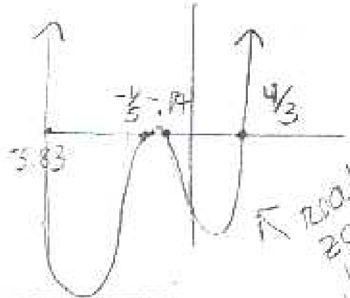
$(3x-4)(5x+1)(x^2+6x+1)=0$

$x = \dfrac{-6 \pm \sqrt{6^2-4(1)(1)}}{2(1)}$

$x = \dfrac{-6 \pm \sqrt{32}}{2} = \dfrac{-6 \pm 4\sqrt{2}}{2} = -3 \pm 2\sqrt{2}$

$x = 4/3, -1/5, -3+2\sqrt{2}, -3-2\sqrt{2}$

$\approx -.17, -5.83$



really zoom in to see! Xmin -.5 Xmax 1

39. The table shows the number of U.S. drive-in movie theaters for the years 1995 to 2002.

Years since 1995	0	1	2	3	4	5	6	7
Drive in movie theaters	848	826	815	750	737	667	663	634

a) Find a cubic function on the graphing calculator that fits this data. Round each value to 4 decimal places.

$y = .9899x^3 - 10.50x^2 - 5.004x + 846.77$

b) If this pattern continues, what will be the number of movie theaters in 2010?

$x = 15$

$y = 1749.88$ $\approx$ 1750 drive-ins

c) In how many years from 1995 will the number of drive in movie theaters reach 1000?

12.09 years from 1995

40. Simplify each radical expression. Show all steps and circle your final answer.

a) $\sqrt{275x^{10}y^7} = \sqrt{25x^{10}y^6}\sqrt{11y} = 5x^5y^3\sqrt{11y}$

b) $\sqrt[3]{135x^5y^{18}} = \sqrt[3]{27x^3y^{18}}\sqrt[3]{5x^2} = 3xy^6\sqrt[3]{5x^2}$

c) $\sqrt[4]{96x^{11}y^{17}} = \sqrt[4]{16x^8y^{16}}\sqrt[4]{6x^3y} = 2x^2y^4\sqrt[4]{6x^3y}$

d) $9\sqrt[3]{5} - 3\sqrt[4]{11} - 5\sqrt[4]{11} - 7\sqrt[3]{5} = 2\sqrt[3]{5} - 8\sqrt[4]{11}$

e) $7\sqrt[3]{54} + 5\sqrt[3]{24} - 9\sqrt[3]{250} - 4\sqrt[3]{192}$

$= 7\sqrt[3]{27}\sqrt[3]{2} + 5\sqrt[3]{8}\sqrt[3]{3} - 9\sqrt[3]{125}\sqrt[3]{2} - 4\sqrt[3]{64}\sqrt[3]{3}$

$= 21\sqrt[3]{2} + 10\sqrt[3]{3} - 45\sqrt[3]{2} - 16\sqrt[3]{3}$

$= -24\sqrt[3]{2} - 6\sqrt[3]{3}$