Series continued, convergence, divergence

Warm ups

(5) $\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n} =$

(6) Find a closed form for

$2+6x+18x^2+54x^3+\ldots$

and determine where (for what values of x) it converges and what it converges to.

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Convergence/Divergence of a ***Sequence***

A sequence $\{a_n\}$ ***converges***

Formally stated:

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Converge or Diverge?

- $\{3n/(n+2)\}_{n=1}^{\infty}$ ....

- $\{ 1/n \}_{n=1}^{\infty}$ ....

- $\{ (-1)^n \}_{n=0}^{\infty}$ ....

- $\{ n \}_{n=1}^{\infty}$ ....

- $\{ \tan^{-1} n \}_{n=1}^{\infty}$ ....

- $\{(2n^2 + 5n - 7)/(5n^2 + 3n + 4)\}_{n=1}^{\infty}$ ....

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Convergence of a *Series*

A Series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots a_n + \ldots$ **Converges**

if and only if

- **If a series** $\sum_{n=1}^{\infty} a_n$ **CONVERGES,**

  and we call the number S that the series converges to the "sum of the series"

- **If a series** $\sum_{n=1}^{\infty} a_n$ **does NOT converge**

  then we say the series DIVERGES

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Examples of ***Series*** Convergence

$$\sum_{n=0}^{\infty} \frac{1}{(2^n)}$$

$$\sum_{k=1}^{\infty} k$$

$$\sum_{n=0}^{\infty} (-1)^n$$

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Convergence Properties of Series

Linearity If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge

then (i) $\sum_{n=1}^{\infty} (a_n + b_n) =$ and

(ii) $\sum_{n=1}^{\infty} ka_n =$ (k being any constant)

Truncation Altering a series by a finite number of terms does NOT affect convergence or divergence (for a convergent series, however, it may affect the sum of the series)

Rearrangement or Regrouping

If a series has all positive terms and it converges, then rearranging or regrouping the terms has no affect on convergence...the series will still converge to the same sum

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n^{th} term test (for divergence)

Statement of theorem:

If a series $\sum_{n=1}^{\infty} a_n$ **converges, then**

More Useful Form (Contrapositive Equivalent):

If **then**

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EXAMPLES:

$\sum_{n=1}^{\infty} \tan^{-1} n$

$\sum_{j=5}^{\infty} 2j/(3j-7)$

Does $\sum_{n=1}^{\infty} 1/n$

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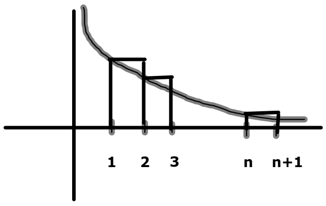
HARMONIC SERIES IS

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

Reason: Look at the continuous curve $f(x) = 1/x$ and compare the area under that curve over $[1, n+1]$ with $s_n = 1 + 1/2 + \ldots 1/n$



$\int_1^{n+1} 1/x \, dx =$) $\underbrace{1 + 1/2 + 1/3 + \ldots + 1/n}_{s_n}$

$\lim_{n \to \infty} \ln(n+1) =$

so $\lim_{n \to \infty} s_n =$ $\therefore$ series

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INTEGRAL TEST

Suppose f is { continuous, positive-valued, decreasing } function on $[c, \infty)$

then the infinite series $\sum_{n=c}^{\infty} f(n)$

does whatever the improper integral $\int_c^{\infty} f(x)\, dx$ does

that is, they <u>both</u> converge (not necessarily to the same number), or they <u>both</u> diverge

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An important consequence of the INTEGRAL TEST

p - series $\sum_{n=1}^{\infty} \frac{1}{n^p}$

$1/(n^p)$

Recall p-integrals? $\int_1^{\infty} \frac{1}{x^p}\,dx$ { converge if ___ ; diverge if ___ }

p - series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ { converge if ___ ; diverge if ___ }

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EXAMPLES: p-series, integral test

(1) $\sum_{n=5}^{\infty} \frac{1}{n^3}$

(2) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

(3) $\sum_{n=1}^{\infty} \frac{1}{n^{1.001}}$

(4) $\sum_{n=2}^{\infty} \frac{1}{(n \cdot \ln(n))}$

(must use integral test)

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REVIEW

We now know:

Geometric Series $\sum_{n=0}^{\infty} a \cdot r^n$

p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$

Harmonic Series $\sum_{n=1}^{\infty} \frac{1}{n}$

2 Tests for convergence/divergence:

- **n^{th} term test**
- **integral test**

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DO 9.2 in your text

#1,4,6,7,9,11,12-15,19,21,23,24,31

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