C. Lucas--Calculus Notetaker/Worker______________

HMWK before chap 3 Shortcuts to differentiation! Completely Manual…**no calculator.**

Remember to always think graphically!
Also, as usual, it really helps to know some of these properties IN WORDS (so you are versatile in their application).

1) $D_x c =$ __0__ ("The derivative of a constant function is _____". What could a constant look like?)

2) Use the limit definition of derivative to show that $D_x(mx+b) = m$

$$D_x(mx+b) = \lim_{a\to x} \frac{(ma+b)-(mx+b)}{a-x} = \lim_{a\to x}\left(\frac{ma-mx}{a-x}\right) = \lim_{a\to x} \frac{m(a-x)}{a-x} = m$$

3) $D_x w^r =$ __$r \cdot w^{r-1}$__ (Ask yourself what type of function this is, and *then* ask yourself how to differentiate it)

power

4) $D_x cf(x) =$ __$c \cdot D_x f(x)$__ Why? (show using limit definition of derivative)

$c \cdot f'(x)$

$$D_x cf(x) = \lim_{a\to x} \frac{cf(x)-cf(a)}{x-a}$$

$$= \lim_{a\to x} \frac{c(f(x)-f(a))}{x-a}$$

$$= c\lim_{a\to x} \frac{f(x)-f(a)}{x-a}$$

$$= c \cdot D_x f(x)$$

5) $D_x(f(x) \pm g(x)) =$ $D_x f(x) \pm D_x g(x)$

6) $D_x(f(x) * g(x)) \neq$ $D_x f(x) \cdot D_x g(x)$ Verify with an example using polynomials or power functions that you know. *(take a guess)*

Let $f(x)=x^2$ $g(x)=x^3$

Then $(f\cdot g)(x)=x^5$

$D_x(f\cdot g)(x)=5x^4$

NO $\stackrel{?}{=} f'(x)\cdot g'(x)$

NO $\stackrel{?}{=} 2x \cdot 3x^2$

$\boxed{\text{The deriv of a product}}$ is ~~not~~ not the prod. of derivatives!

7) $D_r((r^2-1)(r^2+1)) =$ $4r^3$

$D_r(r^4-1)$

8) If s(t) = $3t^2+5t-7$, determine v (2) $=17$

$v(t)=6t+5$

9) Find the slope of the tangent line to the curve y= $3x^{-2}+x^2$ at the point where x=1 $y=4-4(x-1)$

$y'(x)=-6x^{-3}+2x$ $y'(1)=-4$

$(1,4)$

10) Find the point slope equation of the normal line (perpendicular line) to y= $\sqrt{x}$ at point (4,2) (hint: write the square root as a power).

$y'(x)=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$

$y'(4)=\frac{1}{4}$

slope of tan line

slope of normal line is $-4$

$\perp$

$y=2-4(x-4)$

$$D_x[f(x)\cdot g(x)] \neq D_x f(x) \cdot D_x g(x)$$

$$D_x[x^2 \cdot x^3] \stackrel{?}{=} D_x x^2 \cdot D_x x^3$$

$$D_x(x^5) \qquad 2x \cdot 3x^2$$

$$5x^4 \neq 6x^3$$

11) At what point or points does r(x) = $x^2 + 2x$ have horizontal tangents?

$(-1,-1)$

$r'(x) = 2x+2$

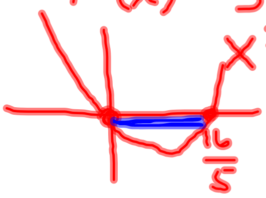
$= 2(x+1) = 0$

$x = -1$

12) Over what intervals is f(x) = $x^5 - 4x^4$ both decreasing and concave down?

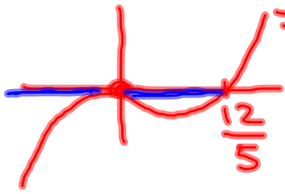
$f'(x) = 5x^4 - 16x^3 < 0$

$x^3(5x-16) < 0$




$f''(x) = 20x^3 - 48x^2 < 0$

$= 4x^2(5x-12) < 0$



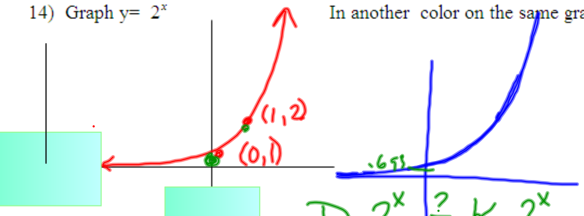

$\left(0, \frac{12}{5}\right)$

13) $D_x 2^x =$ ____________

*(take a guess)*

# Derivatives of EXPONENTIAL FUNCTIONS

14) Graph y= $2^x$ In another color on the same graph, graph y'(x)




The derivative of an exponential functions appears to be exponential.

$D_x 2^x \stackrel{?}{=} k \cdot 2^x$

15) So, it looks like the derivative of an exponential is ______________ Does this agree with what you wrote in #13?__________Why or why not?

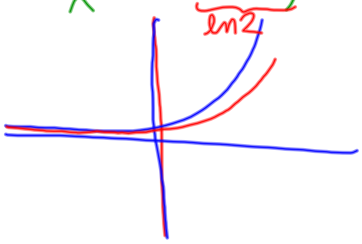
where $|k| < 1$

16) Now, you can take out your TI-89, **only** to complete the following charts. Use h=0.01

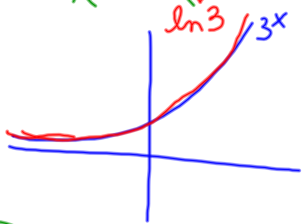
| x | $2^x$ | slopes of tiny secant lines $\frac{2^{x+h}-2^x}{h}$ | $\frac{\frac{2^{x+h}-2^x}{h}}{2^x}$ | x | $3^x$ | $\frac{3^{x+h}-3^x}{h}$ | $\frac{\frac{3^{x+h}-3^x}{h}}{3^x}$ |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 0.695 | 0.695 | 0 | 1 | 1.104 | 1.104 |
| 1 | 2 | 1.391 | 0.695 | 1 | 3 | 3.314 | 1.104 |
| 2 | 4 | 2.782 | 0.695 | 2 | 9 | 9.942 | 1.104 |
| 3 | 8 | 5.564 | 0.695 | 3 | 27 | 29.826 | 1.104 |

$D_x 2^x \approx \underbrace{(0.69)}_{\ln 2} 2^x$

$D_x 3^x \approx \underbrace{(1.104)}_{\ln 3} 3^x$



Derivative below




Derivative above

For what base does the derivative lie right on the function itself?

$D_x \pi^x = (\ln \pi)\pi^x$

$e$

$D_x e^x = \underbrace{1}_{\ln e} \cdot e^x$

$$\boxed{D_x a^x = a^x \cdot \ln a}$$

$$\boxed{D_x e^x = e^x}$$

$D_x 2 \cdot 5^x = 2 D_x 5^x$

$= 2 \cdot 5^x \ln 5$

So...big conclusions for today:

$$D_x 2^x = 2^x \cdot \ln 2$$

$$D_x 3^x = 3^x \cdot \ln 3$$

$$\boxed{\begin{aligned} D_x a^x &= a^x \cdot \ln a \\ D_x e^x &= e^x \end{aligned}}$$

Add to your formula sheet

Many quantities have growth rates that are proportional to the quantities themselves. What does this mean mathematically?

Differentiate if you can:

1) $f(x) = 3^x + 5 \cdot 2^x$

$f'(x) = 3^x \ln 3 + 5 \cdot 2^x \ln 2$

2) $g(x) = \frac{3^x}{4}$ $g'(x) = \frac{1}{4} \cdot 3^x \ln 3$

3) $h(x) = \frac{3^x}{x}$ NO

4) $f(x) = \frac{7^x}{7} - \frac{21}{\sqrt{x}}$

5) $g(p) = e^{\pi} - 4e^{p}$

6) $h(m) = (\ln 2)^m$

7) $r(a) = 4^{2a}$

HOMEWORK:
3.2
EXERCISES