NATURE OF INFINITE SERIES

An Infinite Series

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots + a_n + \ldots$$

consists of two sequences:

- **a term sequence:**

$$\{a_n\}_{n=1}^{\infty} = a_1,\ a_2,\ a_3, \ldots\ ,a_n, \ldots$$

- **a partial sum sequence:**

$$\{s_n\}_{n=1}^{\infty} = \underbrace{a_1}_{s_1}, \underbrace{a_1 + a_2}_{s_2}, \underbrace{a_1 + a_2 + a_3}_{s_3}, \ldots$$

$$s_n = a_1 + a_2 + a_3 + \ldots + a_n$$

Examples (Infinite Series)

$$\sum_{n=1}^{\infty} n = 1 + 2 + 3 + 4 + \ldots + n + \ldots$$

- terms: [redacted]
- partial sums: [redacted]

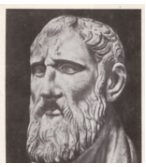
$$\sum_{n=0}^{\infty} \left(\frac{1}{2^n}\right) = 1 + 1/2 + 1/4 + 1/8 + \ldots\ 1/2^n + \ldots$$

- terms: [redacted]
- partial sums: [redacted]

{Sn}=1/2,1/3,1/4,1/5,...

Determine the first four terms of the series having these partial sums.

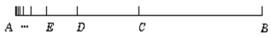
GEOMETRIC SERIES



Zeno of Elea (490 - 435 B.C.)

Zeno's Racecourse Paradox

Suppose a runner has to travel form the start point A to the finish point B. But firstly he has to travel to the midpoint C and thence to B. But if D is the midpoint of AC, he must first travel to D and so on ad infinitum. So since in finite time it is impossible to accomplish an infinite number of movements then the runner is not able to finish his distance.



Why is this a problem? Because the same argument can be made about **half** of the race course: it can be divided in half in the same way t the entire race course can be divided in half. And so can the half of the half of the half, and so on, *ad infinitum*.

Geometric Series: $\sum_{n=0}^{\infty}$ [ ] = [ ]

Sum of first n terms: s_n = [ ]

= [ ]

= [ ]

Sum of series (if |common ratio| < 1):

S = [ ]

= [ ]

Memorize this in Words!!!

Proof of Geometric Sums

$S_n=a_1 + a_2 + a_3 + a_4 +...+ a_n$

$rS_n=ra_1 + ra_2 + ra_3 + ra_4 +...+ ra_n$

so...

$S_n-rS_n=$

so...

$S_n=$

Now what if abs(r)<1 and n-->∞?

Write the following series in sigma (closed) form. Then find their sums, if they exist:

a) 5+10+20+40+80+...

b) 4+7+10+13+16+...

c) 48+24+12+6+3+...

d) 10+1+0.1+0.01+0.001+...

Practice (find the sum if it exists):

(1) $\sum_{n=0}^{\infty} \left(\frac{1}{10}\right)^n$

(2) $\sum_{n=0}^{\infty} 5\left(-\frac{1}{4}\right)^n$

(3) $\sum_{n=1}^{\infty} \frac{3}{10^n}$

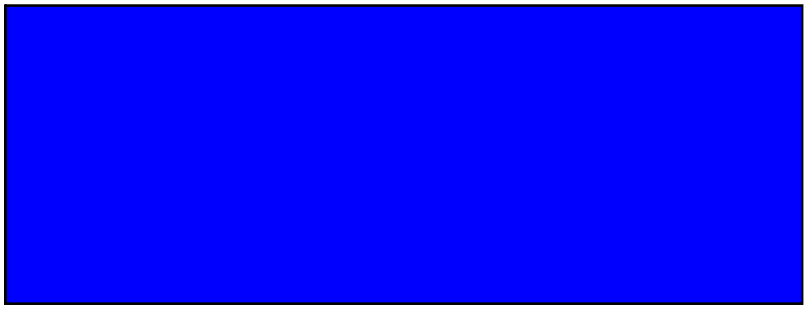
(4) What is $0.\overline{9}$..... ?

What about 0.373737... ?

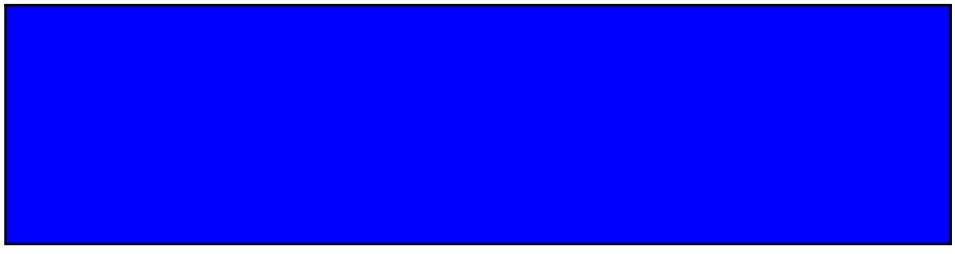
(5) $\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n}$

Convergence/Divergence of a *Sequence*

A sequence $\{a_n\}$ *converges*



Formally stated:



Converge or Diverge?

- $\{3n/(n+2)\}_{n=1}^{\infty}$ .....
- $\{1/n\}_{n=1}^{\infty}$ .....
- $\{(-1)^n\}_{n=0}^{\infty}$ .....
- $\{n\}_{n=1}^{\infty}$ .....
- $\{\tan^{-1} n\}_{n=1}^{\infty}$ .....
- $\{(2n^2 + 5n - 7)/(5n^2 + 3n + 4)\}_{n=1}^{\infty}$ .....

Convergence of a *Series*

A Series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots a_n + \ldots$ Converges

if and only if its [redacted]

- If a series $\sum_{n=1}^{\infty} a_n$ CONVERGES, [redacted]

and we
the seri
"sum of

- If a series $\sum_{n=1}^{\infty} a_n$ does NOT converge [redacted]

then we say the series DIVERGES

Examples of *Series* Convergence

$$\sum_{n=0}^{\infty} \frac{1}{(2^n)}$$

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$$\sum_{k=1}^{\infty} k$$

slide box

$$\sum_{n=0}^{\infty} (-1)^n$$

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