Differentiating Implicitly Defined Functions

WHAT ARE IMPLICITLY DEFINED FUNCTIONS AGAIN?

Definition. A relation $F(x,y) = 0$ is said to **define the function $y = f(x)$ implicitly** if, for x in the domain of f, $F(x,f(x)) = 0$.

Even though we may not be able to determine an explicit function for f, we can determine the derivative of f. How?

When will we use implicit differentiation?

$2x - 3y = 9$ Find $D_x y$

Implicitly defined function

explicitly defined function → $y = \frac{2}{3}x - 3$

$D_x y = \frac{2}{3}$

understand that one variable is a function of the other one

(we'll treat y as a function of x since we're asked to find $D_x y$)

So wherever you see a y, in the expression, treat it like a "blob in x", and use the chain rule.

$2x - 3y = 9$ $\quad 2x - 3(\text{some blob in } x) = 9$

Find $D_x y$

$2 - 3\frac{dy}{dx} = 0$

$D_x y = \frac{dy}{dx} = \frac{2}{3}$

$x^2 y + y^3 - x^3 = 5$ Find $D_x y$

$x^2 \cdot \frac{dy}{dx} + y \cdot 2x + 3y^2 \cdot \frac{dy}{dx} - 3x^2 = 0$

$\frac{dy}{dx}\left(x^2 + 3y^2\right) = 3x^2 - 2xy$

$\frac{dy}{dx} = \frac{3x^2 - 2xy}{x^2 + 3y^2}$

We can use this to determine the slope at any point (x, y) that is on the function.

we need both x + y

$(-\sqrt[3]{5}, 0)$

$(0, \sqrt[3]{5})$

$x^2y+y^3-x^3=5$ Find $D_y x$!

y is independent variable

$$x^2\cdot 1+y\cdot 2x\cdot\frac{dx}{dy}+3y^2-3x^2\frac{dx}{dy}=0$$

$$\frac{dx}{dy}\left(2xy-3x^2\right)=-3y^2-x^2$$

$$\frac{dx}{dy}=\frac{-3y^2-x^2}{2xy-3x^2}=\frac{\cancel{-}(3y^2+x^2)}{\cancel{-}(3x^2-2xy)}$$

Compare

$$\frac{dy}{dx}=\frac{3x^2-2xy}{x^2+3y^2}\qquad \frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}$$

Reciprocals

HOMEWORK :

SHEET...BOTH SIDES,
and 3.7 # 1,2,7,9,11

QUIZ CHANGED TO MONDAY

Attachments

- Visual Calculus Implicit Differentiation
- Implicit Function Graphing