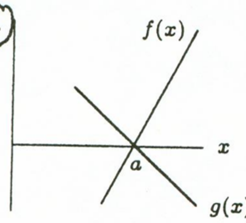
Exercises and Problems for Section 3.10

Exercises

For Exercises 1–4, find the sign of $\lim_{x\to a}\dfrac{f(x)}{g(x)}$ from the figure.

1.



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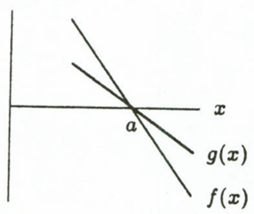

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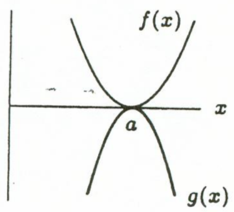

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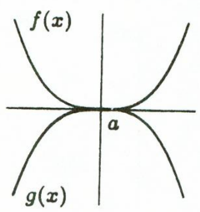


Based on your knowledge of the behavior of the numerator and denominator, predict the value of the limits in Exercises 5–8. Then find each limit using l'Hopital's rule.

5. $\lim_{x\to 0}\dfrac{x^2}{\sin x}$

6. $\lim_{x\to 0}\dfrac{\sin^2 x}{x}$

7. $\lim_{x\to 0}\dfrac{\sin x}{x^{1/3}}$

8. $\lim_{x\to 0}\dfrac{x}{(\sin x)^{1/3}}$

In Exercises 9–12, which function dominates as $x\to\infty$?

9. x^5 and $0.1x^7$

10. $0.01x^3$ and $50x^2$

11. $\ln(x+3)$ and $x^{0.2}$

12. x^{10} and $e^{0.1x}$

Problems

13. Evaluate $\lim_{x\to 0^+} x\ln x$. [Hint: Write $x\ln x = \dfrac{\ln x}{1/x}$.]

14. (a) What is the slope of $f(x)=\sin(3x)$ at $x=0$?
(b) What is the slope of $g(x)=5x$ at $x=0$?
(c) Use the results of parts (a) and (b) to calculate $\lim_{x\to 0}\dfrac{\sin(3x)}{5x}$.

Explain why l'Hopital's rule cannot be used to calculate the limits in Problems 15–17. Then evaluate the limit if it exists.

15. $\lim_{x\to 1}\dfrac{\sin(2x)}{x}$

16. $\lim_{x\to 0}\dfrac{\cos x}{x}$

17. $\lim_{x\to\infty}\dfrac{e^{-x}}{\sin x}$

18. Find the horizontal asymptote of $f(x)=\dfrac{2x^3+5x^2}{3x^3-1}$.

19. The functions f and g and their tangent lines at $(4,0)$ are shown in Figure 3.53. Find $\lim_{x\to 4}\dfrac{f(x)}{g(x)}$.

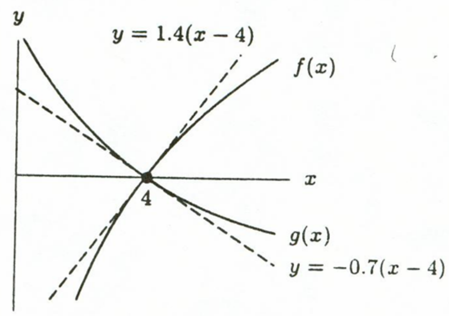


Figure 3.53

Over

Calculus
L'Hopital assignment page 2

L'Hopitaler______________

Adapted from Swokowski Calculus section 11.1

Find the following limits:

1) $\lim\limits_{x\to 0}\dfrac{\sin(x)}{2x}$

2) $\lim\limits_{x\to 0}\dfrac{5x}{\tan(x)}$

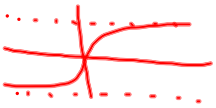
3) $\lim\limits_{x\to 2}\dfrac{2x^2-5x+2}{5x^2-7x-6}$

4) $\lim\limits_{x\to 1}\dfrac{x^3-3x+2}{x^2-2x-1}$

5) $\lim\limits_{x\to 0}\dfrac{x-\sin(x)}{x^3}$

6) $\lim\limits_{x\to \infty}\dfrac{x^2}{\ln(x)}$

7) $\lim\limits_{x\to 0}\dfrac{2x}{\tan^{-1}x} \overset{L'H}{=}$



$\lim\limits_{x\to 0}\dfrac{2}{\frac{1}{x^2+1}} = 2$

8) $\lim\limits_{x\to \infty}\dfrac{2x^2+3x+1}{5x^2+x+4}$

9) $\lim_{x \to 2^+} \frac{\ln(x-1)}{(x-2)^2}$

$$\lim_{x \to 2^+} \frac{\frac{1}{x-1}}{2(x-2)} = \text{DNE}$$

$\frac{1}{x-1} \to 1$, $2(x-2) \to 0$

10) $\lim_{x \to \infty} \frac{\ln(\ln x)}{\ln x}$

11) $\lim_{x \to 0^+} x \ln x$

12) $\lim_{x \to \infty} x(e^{\frac{1}{x}} - 1)$

$$\lim_{x \to \infty} \frac{e^{\frac{1}{x}} - 1}{\frac{1}{x}}$$

$$\lim_{x \to \infty} \frac{e^{\frac{1}{x}} \cdot -\frac{1}{x^2}}{-\frac{1}{x^2}}$$

1

13) $\lim_{x \to \infty} (e^{-x} \ln x)$

14) $\lim_{x \to 0^+} x^x$

15) $\lim_{x\to\infty} x^{\frac{1}{x}} = \lim_{x\to\infty} e^{\frac{1}{x}\ln x}$

$= e^{\lim_{x\to\infty}\frac{\ln x}{x}} \overset{L'H}{=} e^{\lim_{x\to\infty}\frac{\frac{1}{x}}{1}} = 1$

16) $\lim_{x\to 0^+}(2x+1)^{\cot x}$

17) $\lim_{x\to\infty}\left(\frac{x^2}{x-1}-\frac{x^2}{x+1}\right)$

18) $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$

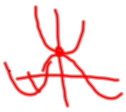
19) $\lim_{x\to 1^-}(1-x)^{\ln x}$

20) $\lim_{x\to\frac{\pi}{2}^-}(1+\cos x)^{\tan x}$

16) $\lim_{x\to 0^+} e^{\cot x\ln(2x+1)}$

$e^{\lim_{x\to 0^+}\frac{\ln(2x+1)}{\tan x}}$

$\overset{L'H}{=} e^{\lim_{x\to 0^+}\frac{\frac{1}{2x+1}\cdot 2}{\sec^2 x}}$ → 2, → 1



e^2

18) $\lim_{x\to 0}\frac{\sin x - x}{x\sin x}$

$\overset{L'H}{=} \lim_{x\to 0}\frac{\cos x - 1}{x\cos x+\sin x}$

$\overset{L'H}{=} \lim_{x\to 0}\frac{-\sin x}{-x\sin x+\cos x+\cos x}$

$= \frac{0}{2}$

19) $\lim_{x\to 1^-}(1-x)^{\ln x}$

$\lim_{x\to 1^-} e^{\ln x\ln(1-x)}$

$e^{\lim_{x\to 1^-}\ln x(\ln(1-x))}$

$e^{\lim_{x\to 1^-}\frac{\ln(1-x)}{\frac{1}{\ln x}}}$

$\overset{L'H}{=} e^{\lim_{x\to 1^-}\frac{\frac{1}{1-x}\cdot -1}{-(\ln x)^{-2}\frac{1}{x}}} = e^{\lim_{x\to 1^-}\frac{\frac{1}{x-1}}{\frac{-(\ln x)^{2}}{x}}}$

$\overset{L'H}{=} e^{\lim_{x\to 1^-}\frac{x(\ln x)^2}{(1-x)}}$

$\overset{L'H}{=} e^{\lim_{x\to 1^-}\frac{\cancel{x}\cdot 2(\ln x)\cdot\frac{1}{\cancel{x}}}{-1}}$

$= e^0 = 1$

L'Hopital Exercises

#20) $\lim_{x\to\frac{\pi}{2}^-}(1+\cos x)^{\tan x} = \lim_{x\to\frac{\pi}{2}^-} e^{\tan x \cdot \ln(1+\cos x)}$

$$= e^{\lim_{x\to\frac{\pi}{2}^-} \frac{\ln(1+\cos x)}{\cot x}}$$

$\frac{0}{0}$ type L'H

$$= e^{\lim_{x\to\frac{\pi}{2}^-} \frac{\frac{1}{1+\cos x}\cdot -\sin x}{-\csc^2 x}}$$

$$= e^{\lim_{x\to\frac{\pi}{2}^-}\left(\frac{\sin^3 x}{1+\cos x}\right)}$$

$$= e^{1} = e$$

$$\boxed{y = \log_{10} x = \frac{\ln x}{\ln 10}}$$

$\log x = y$

$10^y = x$

$D_x y = ?$

$D_x \frac{\ln x}{\ln 10} = \frac{1}{\ln 10}\cdot\frac{1}{x}$

$D_x(10^y = x)$

$$\boxed{10^y \cdot \ln 10 \frac{dy}{dx} = 1 \qquad \frac{dy}{dx} = \frac{1}{10^y \cdot \ln 10} = \frac{1}{x\cdot \ln 10}} = \frac{\log e}{x}$$