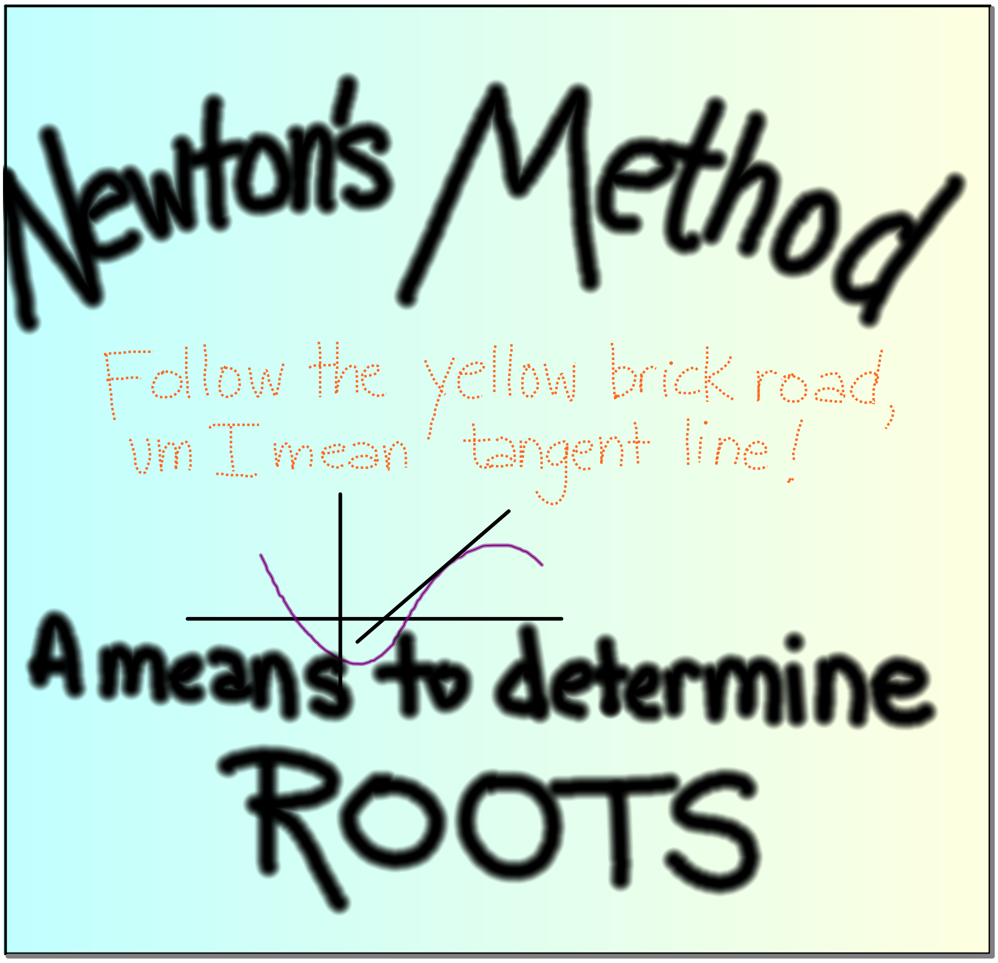
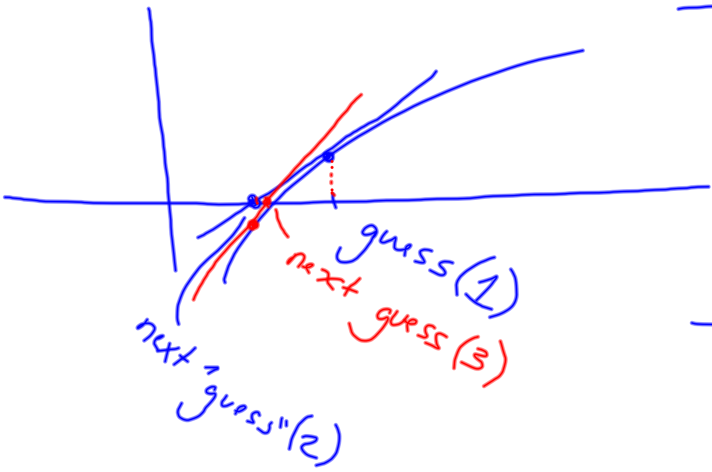


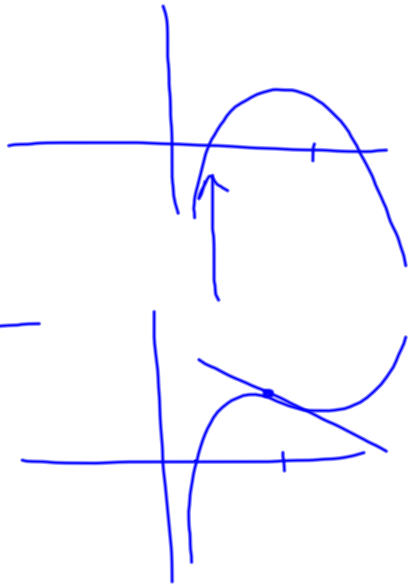
C. Lucas---Calculus
Newton's Method---Another Application of the Derivative

HOMEWORK: *Appendix D # 1 ,2, 5 ,7, 11*

"Follow the Tangent Line"…Newton's method is an iterative (repetetive process) means of determining roots. Since it is iterative, it is very suited for computers and calculators.

Here's the development:





Suppose for the graph above, you wish to determine the root to a certain number of decimal places (you will stop the iterations, by the way, when your results agree to that number of places).

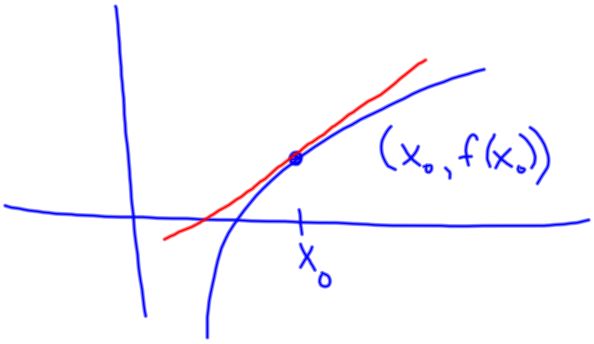
1) Take a guess at the root, call it x_0

2) Now find $f(x_0)$, and draw a tangent line to the graph of f at $(x_0, f(x_0))$

3) Since you have a point and the slope, you can write the equation of this tangent line. Do so here:

$f'(x_0)$

$$y = f(x_0) + f'(x_0)(x - x_0)$$



4) Now, if your drawn line does not already intersect the x-axis, extend it so that it does.

5) The place where your tangent line intersects the x-axis will be your 'new' guess at the root. It should be closer to the actual root than your previous guess was. Label it x_1 on your drawing.

6) The full coordinates of the point you just labeled are (x_1 , 0)

7) Now, since this point is on your tangent line, it should fit into your equation that you wrote in number 3 above. Put this point into your equation.

$$y = f(x_0) + f'(x_0)(x - x_0)$$

$$0 = f(x_0) + f'(x_0)(x_1 - x_0)$$

solve for x_1

$$\frac{-f(x_0)}{f'(x_0)} = x_1 - x_0$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

7) Now, since this point is on your tangent line, it should fit into your equation that you wrote in number 3 above. Put this point into your equation.

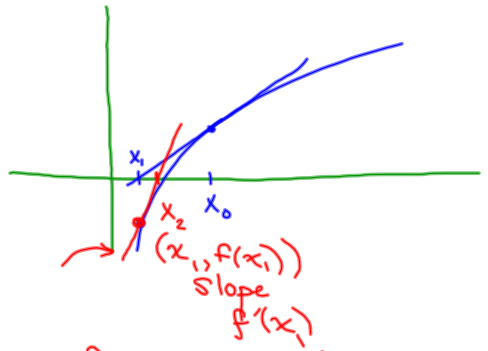
_____ = ____

8) Now, since x_1 is a better approximation to the actual root, we are interested in finding it. Take the equation you wrote in #7 and solve it for x_1 (it will be in terms of f, f', and x_0)

x_1 = ______________

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

8) Now, you will repeat the process. Draw a tangent line to f at $(x_1, f(x_1))$, write the equation of this line, extend it to see where it intersects the x-axis, label that point $\underline{x_2}$, put it in your equation and solve for it.



$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$y = f(x_1) + f'(x_1)(x - x_1)$

$(x_2, 0)$ is a point on this

$0 = f(x_1) + f'(x_1)(x_2 - x_1)$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

9) Now, generalize. If you have the n^{th} guess at a root, and wish to find the $(n+1)^{st}$, what formula would you use?

Newton's Method:

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

10)Now practice. Using Newton's method, find the following correct to four decimal places: (Show a table of your iterations).

a) The largest root of $f(x)= x^5-7x^4+2x^3-10x+3$

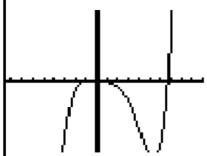
b) $\sqrt[4]{100}$ → the func. is $y = x^4 - 100$

a)

n	x_n
0	7
1	6.76946
2	6.73502
3	6.7343
4	6.73429
5	6.73429

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$7 - \frac{f(7)}{f'(7)}$$



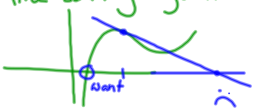
$y = x^4 - 100$

b) guess. $x_0 = 3$

n	x_n
0	3
1	3.17592
2	3.16236
3	3.162277
4	3.162277

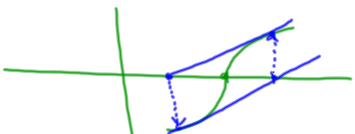
What could go wrong?

1) Guess is near local extrema + tangent line sends you farther away.



or is at a local extrema:

2) Graph is "odd / origin" symmetric about its root



3) Tangent line sends you out of the domain