Precalculus Learning Targets

What follows is what a student in Precalculus should know and be able to do by the end of each semester. It is broken up by unit of study and also includes what portion of the textbook, *Precalculus: Graphical, Numerical, Algebraic*, is used in each unit. Since mathematics is cumulative in nature, many learning targets are expected to be used not only in the unit where they are introduced, but throughout the course. This list is published as a courtesy to all interested learners, and is not intended for use outside of this specific course.

<u>First Semester</u>:

A. Precalculus Essentials (Appendix A, Preliminary Chapter P)

1. Apply slope and the distance formula to solve problems involving the Cartesian coordinate system.
2. Add, subtract, multiply, or divide algebraic fractions.
3. Simplify complex algebraic fractions (a fraction where the numerator and/or denominator are also fractions).
4. Solve quadratic and quadratic-like equations algebraically. (Methods include factoring, using square roots, and applying the quadratic formula.)
5. Model application problems using a quadratic.
6. Solve quadratic inequalities using a sign chart.

B. Functions and Transformations (Chapter 1, sections 1-5)

1. Recognize eight parent functions (linear, quadratic, absolute value, square root, cubic, cube root, inverse, inverse square).
2. Solve equations or inequalities involving the parent functions graphically using intersection or zeros (roots) both by hand and using the graphing calculator.
3. Solve a system of equations involving the parent functions algebraically or graphically (both by hand and using the graphing calculator).
4. Use function notation. Explain the real-life meaning of function notation.
5. Identify transformations as inside or outside, shifts, stretches/compressions, or reflections over the *x*- or *y*- axes.
6. State the domain and range of a function using interval notation.
7. Find a function's extrema (local maximum or minimum). Identify the *x*-intervals where a function is increasing, decreasing, or constant.
8. Use symmetry to identify a graph as even, odd or neither.
9. Work with piece-wise functions (graph, write the equation given the graph, evaluate *f*(n)).
10. Without graphing, determine if a piece-wise function is continuous.
11. Perform composition of functions algebraically or numerically.
12. Decompose a function into two or three functions.
13. Find the equation of a function's inverse.

C. Modeling with Functions (Sections 1.6, 2.1–2.3)

1. Start with a familiar formula and create a function using substitution (for example, write a formula for the area of a circle as a function of its circumference).
2. Fit a function to data using the appropriate regression on your graphing calculator.
3. Write the equation of a line in slope-intercept form or point-slope form. In a real-life context, explain the meaning of the slope and *y*-intercept.
4. Convert a quadratic function from standard form to vertex form by completing the square.
5. Graph a quadratic function, identifying the zeros (roots), *y*-intercept, vertex, and one other point. Use symmetry to find another point.

6. Write the equation of a quadratic function given the vertex and one other point.
7. Recognize families of power functions in the form $f(x) = a \cdot x^p$ (even positive integer powers, odd positive integer powers, etc.).
8. Graph and analyze a power function (domain, range, increasing, decreasing, symmetry, extrema, vertical asymptote, horizontal asymptote, end behavior using limit notation).
9. Use the ratio method to fit a power function to two points.
10. Recognize that polynomial functions are characterized by degree and have end behavior that is similar to the same degree power function.

D. Optimization

1. Write the equation of a function that is to be maximized or minimized, taking any constraints into consideration.
2. Use your graphing calculator to graph the function, then find the maximum or minimum.
3. Interpret the maximum or minimum in the context of the problem (answer the problem's question).

E. Polynomial and Rational Functions (Chapter 2)

1. Graph a polynomial in factored form, taking into consideration end behavior (degree and leading coefficient) and multiplicities (repeated factors).
2. Use synthetic or long division to determine if $x - k$ is a factor of a polynomial.
3. Use the Fundamental Theorem of Algebra to determine how many zeros a polynomial has.
4. Recognize that if a polynomial has complex zeros, they come in conjugate pairs.
5. Given a "hint," find all remaining zeros of a polynomial. Use synthetic or long division to divide out known zeros, and then use the quadratic formula or factoring to find any remaining zeros.
6. Given the graph, write the equation of a polynomial function in factored form. Use a point on the graph to solve for "a."
7. Determine the end behavior of a rational function (ratio of leading terms). Use limits to describe the end behavior.
8. Determine the equations of any vertical asymptotes and use one-sided limits to describe the behavior near the vertical asymptote(s).
9. Determine the ordered pairs of any holes of a rational function.
10. Graph a rational function; identify the end behavior (horizontal asymptote or slant asymptote), vertical asymptote(s), hole(s), *x*-intercept(s), and/or *y*-intercept.
11. Write a rational function in factored or standard form that meets given conditions.
12. Solve rational equations and inequalities using a sign chart.

F. Exponential, Logarithmic, and Logistic Functions (Chapter 3)

1. Write exponential growth or decay equations in the form $f(x) = a \cdot b^x$ or $P(t) = P_0(1 \pm r)^t$.
2. Recognize an equation as exponential growth or decay, apply any transformations (reflect over the *x*- or *y*-axes), and use limits to describe the end behavior.
3. Compound growth or decay *k* times per year using equations in the form $P(t) = P_0\left(1 \pm \frac{r}{k}\right)^{kt}$.
4. Continuously compound growth or decay using equations in the form $P(t) = P_0 e^{rt}$.
5. Use the ratio method to fit an exponential function to two points.
6. Recognize the shape of a logistic function's graph. Find its initial value and maximum value.
7. Understand that logs are exponents. Be able to switch between exponential and logarithmic form. Solve log problems without a calculator. Be familiar with both common and natural logs.
8. Use the product, quotient, and power properties of logs.
9. Solve exponential, logarithmic and logistic equations algebraically or graphically.

10. Recognize that $f(x) = a \cdot b^x$ and $g(x) = \log_b x$ are inverse functions and undo each other. In other words, $\log_b b^p = p$ and $b^{\log_b n} = n$.
11. Use logs to solve real-life problems involving half life, Newton's Law of Cooling, finance, etc.

G. Logic (Appendix C)

1. Negate universal or existential statements. Use DeMorgan's laws to negate compound statements that involve "and" or "or." Negate a conditional statement.
2. Know the basic truth tables for negation, and, or, conditional, and biconditional.
3. Use a truth table to determine if a statement is a tautology, contradiction, or neither.
4. Use modus ponens, modus tollens, law of contrapositive, law of syllogism, and fallacies of the converse or inverse to identify valid or invalid arguments.
5. Symbolize statements. Use symbols to write direct and indirect proofs.

<u>Second Semester</u>:

H. **Trig (Chapters 4 & 5)**

1. Given a graph that describes one's height on a Ferris wheel as a function of time, be able to describe the Ferris wheel and find the equation that models the situation.
2. Understand the difference between degrees and radians and be able to convert.
3. Find the arc length on a circle.
4. Understand unit circle "basics": positive and negative coterminal angles, sine, cosine, tangent, cotangent, etc.
5. Using symmetry of the unit circle, be able to determine the sine, cosine, etc. of an angle when given one of its trig values.
6. Given the graph of a transformed sine, cosine, or tangent function, write its equation. Given the equation, graph it (determine period, midline, amplitude, horizontal shift).
7. Use a sinusoidal function to model a real-life situation that occurs periodically.
8. Verify a trig identity (use factoring, find a common denominator, etc.).
9. Use double-angle, half-angle, or sum and difference identities to find <u>exact</u> trig values (for example, sin 195°).
10. Find all solutions to a trigonometric equation on a given interval and be able to verify if the solution(s) are correct.

I. Vectors (Sections 5.5, 5.6, 6.1, 6.2, and 8.6)

1. Solve application problems using SOHCAHTOA.
2. Solve an application problem involving vectors by using the Law of Sines and/or Cosines.
3. Resolve a vector (in either 2 or 3 dimensions) into its components.
4. Graph the sum, difference, and scalar multiple of vectors.
5. Determine the magnitude of a vector (in either 2 or 3 dimensions).
6. Know the definition of a unit vector. Write a unit vector in the same direction as the given vector.
7. Use the dot product to determine the angle between vectors in 2 or 3 dimensions.
8. Determine if vectors are parallel, anti-parallel, orthogonal, or neither.

J. Parametric Equations (Section 6.3)

1. Graph a parametric equation by creating a table and describe the motion.
2. Eliminate the parameter in a set of parametric equations.
3. Write parametric equations for circles or ellipses, considering the starting point and rotation.
4. Find the parametric equations that model a real-life situation. Determine when and where the projectile hits the ground, the maximum height of the projectile, etc.

5. Solve a problem involving 2 simultaneous projectiles.
6. Write parametric equations that follow a linear path between 2 points.

K. Sequences and Series (Section 9.4)/ Proof by Induction

1. Find the terms of a sequence from a recursive or an explicit formula.
2. Write a recursive and an explicit formula for arithmetic or geometric sequences.
3. Describe a series using sigma notation and find the sum of a finite arithmetic or geometric series using a formula.
4. Recognize if a geometric series converges or diverges and find the sum of a converging infinite geometric series.
5. Write a proof by induction to show that an explicit formula for a series holds true for all *n*. (e.g., show $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$ for all values of *n*.)
6. Write a proof by induction to show an algebraic expression is divisible by a certain number for all *n*. (e.g., show 3^{n-1} is divisible by 3 for all values of *n*.)

L. Polar Graphing (Sections 6.4 and 6.5)

1. Plot polar coordinates. Give alternate names for the same point.
2. Convert rectangular coordinates to polar and vice versa.
3. Convert equations from rectangular to polar and vice versa.
4. Use an auxiliary graph to create the polar graph.
5. Be familiar with cardioids, limaçons, roses, and spirals (match graph and equation).

M. Rates of Change (Section 10.1)

1. Find the average rate of change (slope of the secant line) over an interval using an equation or graph.
2. Explain the real-life meaning of the average rate of change in an application problem.
3. Analyze a function by finding its domain, range, *x*-intervals where it is increasing, decreasing, concave up, concave down.
4. Sketch a graph that meets given restrictions (e.g., has a specified domain, is even, has a certain vertical asymptote, etc.).
5. Find the instantaneous rate of change using the limit definition of the derivative.
6. Use the derivative to find the slope and equation of the tangent line. Understand the value of the derivative at minimums and maximums of a graph.

N. Limits (Section 10.3)

1. Describe the 3 cases where a limit fails to exist.
2. Evaluate limits (including one-sided limits) graphically (both with and without a calculator).
3. Evaluate a limit numerically (using a table).
4. Evaluate a limit algebraically. (Techniques include: factoring and canceling, finding a common denominator, multiplying by the conjugate.)
5. Understand that the limit as $x \to \pm\infty$ is the same as determining the end behavior of a graph.
6. Understand that a one-sided limit near a vertical asymptote involves either $\pm\infty$.