Review

The derivative of an exponential function is

Differentiate if you can:

1) $f(x) = 3^x + 5 \cdot 2^x$

$f'(x) = (\ln 3)3^x + 5 \cdot 2^x \ln 2$

2) $g(x) = \frac{3^x}{4}$ $\qquad g'(x) = \frac{1}{4} \cdot 3^x \ln 3$

$\cancel{\left(\frac{3}{4}\right)^x}$

3) $h(x) = \frac{3^x}{x}$

NOT YET !

$+4$

$D_x[5 \cdot 2x]$

$5 \underbrace{D_x[2x]}$

$5 \cdot 2$

10

4) $f(x)=\frac{7^x}{7}-\frac{21}{\sqrt{x}}=\frac{7^x}{7}-21x^{-\frac{1}{2}}$

$f'(x)=\frac{7^x\ln 7}{7}+\frac{21}{2}x^{-3/2}$

5) $g(p)=e^{\pi}-4e^{p}$

$g'(p)= \quad -4e^{p}$

6) $h(m)=(\ln 2)^m$

$h'(m)=(\ln(\ln 2))(\ln 2)^m$

7) $r(a)=4^{2a}=(4^2)^a=16^a$

$r'(a)=(\ln 16)(16^a)$

Product & Quotient Rules

$f(x) = x^3$ $\qquad f'(x) = 3x^2$

$g(x) = 2x^5$ $\qquad g'(x) = 10x^4$

$(f \cdot g)(x) = 2x^8$

$10x^7 + 6x^7$

$(f \cdot g)'(x) = 16x^7$ $\qquad f(x) \cdot g'(x) + g(x) f'(x)$

So what's another way to determine $(f \cdot g)'(x)$ from f, g, f' and g'? (since you won't always have the power rule to use!)

$D_x[\text{first} \cdot \text{second}] =$

first · deriv. of second + second · deriv of first

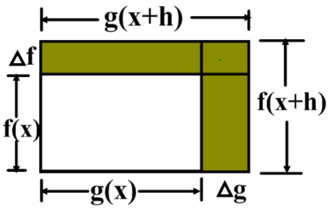
$$(f \cdot g)'(x) =$$

The PRODUCT RULE says (in words) that...

Now for proof of the Product Rule:

$\Delta f = f(x+h) - f(x)$

$\Delta g = g(x+h) - g(x)$



Write two expressions for the shaded region and set them equal.

$$f(x+h)g(x+h) - f(x)g(x) = \Delta f\,\Delta g + f(x)\Delta g + g(x)\Delta f$$

Divide by h and ?

$$\frac{f(x+h)g(x+h) - f(x)g(x)}{h} = \frac{\Delta f\,\Delta g}{h} + \frac{f(x)\Delta g}{h} + \frac{g(x)\Delta f}{h}$$

take limit as $h \to 0$

$$\lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} = \lim_{h\to 0} \frac{\Delta f\,\Delta g}{h} + \lim_{h\to 0} \frac{f(x)\Delta g}{h} + \lim_{h\to 0} \frac{g(x)\Delta f}{h}$$

$$f(x)\lim_{h\to 0}\frac{\Delta g}{h} \qquad g(x)\lim_{h\to 0}\frac{\Delta f}{h}$$

$$f(x)\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$D_x[f(x)g(x)] = f(x)g'(x) + g(x)\cdot f'$$

$(f\cdot g)'(x)$

Quotient Rule

$$Q(x) = \frac{f(x)}{g(x)}$$

Goal: Find Q'(x)

$$Q(x) = \frac{f(x)}{g(x)}$$

goal find $Q'(x)$

$$Q(x) \cdot g(x) = f(x)$$

take deriv.

$$Q(x) \cdot g'(x) + g(x) \cdot Q'(x) = f'(x)$$

$$Q'(x) = \frac{f'(x) - Q(x) g'(x)}{g(x)}$$

$$= \frac{f'(x) - \frac{f(x)}{g(x)} \cdot g'(x)}{g(x)}$$

$$= \frac{\frac{f'(x)}{g(x)} g(x) - \frac{f(x) g'(x)}{g(x)}}{g(x)}$$

$$D_x\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) f'(x) - f(x) g'(x)}{(g(x))^2}$$

The Quotient Rule in words says:

$$\text{Deriv}\left[\frac{\text{num}}{\text{den}}\right] = \frac{\text{den}\cdot\text{deriv num} - \text{num}\cdot\text{deriv den}}{(\text{den})^2}$$

$$D_x\left[\frac{hi}{ho}\right] = \frac{ho\,d\,hi - hi\,d\,ho}{ho\,ho}$$

Determine $D_t(e^t\sqrt{t}) = D_t(e^t t^{\frac{1}{2}})$

$$= e^t\cdot\frac{1}{2}t^{-\frac{1}{2}} + t^{\frac{1}{2}}\cdot e^t$$

If $f(s) = s^3 + 2s$, $g(s) = 3^s$ **then determine** $D_s((f\cdot g)(s))\Big|_{s=2}$

$$D_s(f\cdot g)(s) = (s^3+2s)\cdot(3^s\cdot\ln 3) + 3^s(3s^2+2)$$

at 2... $(12)(9\ln 3) + 9(14)$

$$108\ln 3 + 126$$

The concentration of a drug in the bloodstream follows the function $C(t) = 0.5te^{-t}$ $\to \left(\frac{1}{e}\right)^t$. At three minutes after injection, is the concentration increasing or decreasing and how fast?

$$C'(t) = 0.5\left[t \cdot \left(\frac{1}{e}\right)^t \underbrace{\ln\left(\frac{1}{e}\right)}_{-1} + \left(\frac{1}{e}\right)^t \cdot 1\right]$$

eval. at $t = 3$

is $C'(3) > 0$
or $C'(3) < 0$?

$$e^{-t} = \frac{1}{e^t} = \left(\frac{1}{e}\right)^t$$

Differentiate two different ways: $\dfrac{5^x}{x^3}$

Differentiate: $\dfrac{r^3 + 2r}{r^2 + 5r}$

At what point(s) does $f(x)=\dfrac{x^2}{x^2+1}$

have a horizontal tangent?

Find an equation of the tangent line (in point-slope form) to the graph of

$$g(x)=(x^3-3x+1)(x+2)$$

at point (1,-3).

In case we haven't already answered these!

use roll of masking tape for demo.

What is $D_r A_\odot$ **and why does it make sense?**

What is $D_r V_\odot$ **and why does it make sense?**