Related Rates

Some really nice visuals.

Related Rates

i. Related rate problems frequently involve observing the *rate* of one quantity that is related in some way (frequently via a composition) to the rate of other quantities whose rates are known. (hence the term 'related rates'). Since they deal with *rates*, such situations lend themselves nicely to a calculus application of _derivatives_______! If the relationship between the quantities is in the form of a composition, then, upon differentiating, make sure to use_chain rule_________. And therefore, don't forget that _implicit_________differentiation may make the algebra a LOT easier. So, before plunging into a plethora of differentiation formulas, pause and consider easier routes if they are applicable.

ii. With your group mates, think of a few 'big' situations in our nation or world that may involve related rates (hint: you might consider the one on the other handout to be one example):

rain + crop prod
CO_2 + global temp.
supply + demand.
global temp + polar ice cap.

Many many more (we listed many in class!)

I. Here's a familiar problem you may recall from our last unit. As we set it up, pay attention to the **process and organization of the solution** method.

Suppose air is being pumped into a spherical balloon so that the volume is increasing at a constant rate of $5m^3/min$. At what rate is the radius increasing (with respect to time) at the instant when the radius is 6 meters?

<u>Know/Have</u> <u>Want</u>

$\frac{dV}{dt} = 5\frac{m^3}{min}$ $\frac{dr}{dt}$ when $r=6$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$5 = 4\pi(6)^2 \cdot \frac{dr}{dt}$$

$\frac{m^3}{min}$ m^2

$$\frac{5}{144\pi} \frac{m}{min} = \frac{dr}{dt}$$

IV. If many related rate problems are of the form as the one above, what is a good $\boxed{\textbf{STRATEGY}}$ to follow when setting them up?

1) List quantities that are known + what you want.
(interpret rates as derivatives)

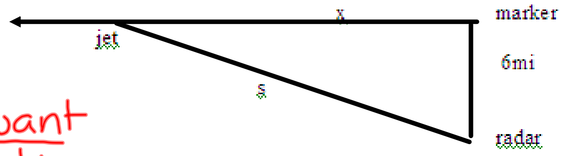
2) Need/write a connecting equation of quantities.

3) Differentiate this connecting equation, oftentimes <u>implicitly</u>.

4) Substitute known quantities + solve for unknown.

Watch + track <u>units</u>.

VII. A low flying jet aircraft covering a straight course is tracked by a radar station set 6 miles to one side of the flight path (see figure). Let x= distance from a reference marker on the course, and s= direct distance from the aircraft to the radar unit. A radar unit can measure only the "range" s and the rate of change ds/dt. Suppose at the time of the radar reading, the jet is 10 miles from the sensor and this distance is increasing at 800mph. Find the actual speed of the aircraft.



Know

$\frac{ds}{dt} = 800$

want

$\frac{dx}{dt}$

when

$s = 10$

or

$x = 8$

$$x^2 + 6^2 = s^2$$

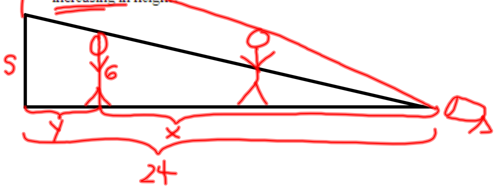
$$\cancel{2}x\frac{dx}{dt} = \cancel{2}s\frac{ds}{dt}$$

$$8\text{mi} \cdot \frac{dx}{dt} = 10\text{mi} \cdot 800\frac{\text{mi}}{\text{hr}}$$

$$\frac{dx}{dt} = 1000\frac{\text{mi}}{\text{hr}}$$

a good demo

III. Here's an example involving a theater (lighting) application:
An off Broadway is exactly 6 feet tall. One night, he delivers a stirring soliloquy on stage. For dramatic effect, he is illuminated only by a single footlight, which is at the same level as the floor of the stage. 24 feet behind the footlight is the vertical backdrop of the stage. As the actor nears the peak of his soliloquy, he begins walking toward the audience (and toward the footlight) at 2 feet per second. When he is 8 feet from the footlight, at what rate is his shadow on the backdrop increasing in height?



Know

$\frac{dx}{dt} = -2$

Want

$\frac{ds}{dt}$ when $x = 8$

$$\frac{6}{x} = \frac{s}{24}$$

$$6x^{-1} = \frac{1}{24}s$$

$$-6x^{-2}\frac{dx}{dt} = \frac{1}{24}\frac{ds}{dt}$$

$$\frac{+12}{64} \cdot 24 = \frac{ds}{dt}$$