The Greatest Integer Function

$$[\![x]\!]$$

$$f(x) = [\![x]\!]$$

What is a function and what does this one do?

$[\![x]\!]$

$[\![3.8]\!] = 3$

$[\![10.99]\!] = 10$

$[\![0.4]\!] = 0$

$[\![-1.2]\!] = -2$

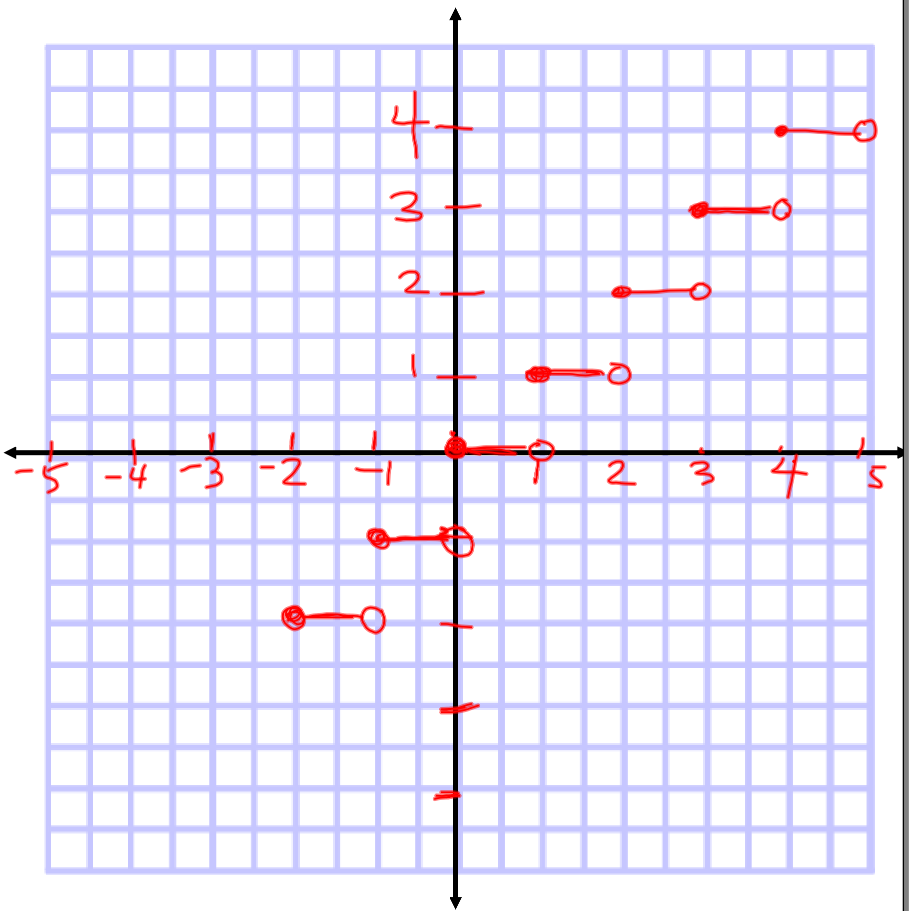
$[\![-0.5]\!] = -1$

$[\![\quad]\!]$

$[\![\quad]\!]$

$[\![\quad]\!]$

$[\![\quad]\!]$

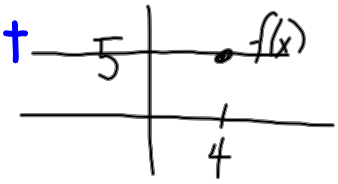


THE LIMIT! WHAT A CONCEPT!

Everything (important) in Calculus is based on LIMITS!

getting infinitely large...
or infinitesimally small...

Intuitive Definition of Limit

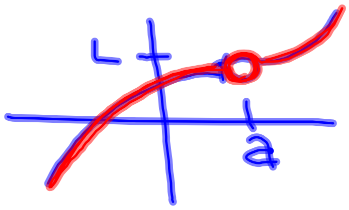


$\lim_{x \to 4} f(x) = 5$

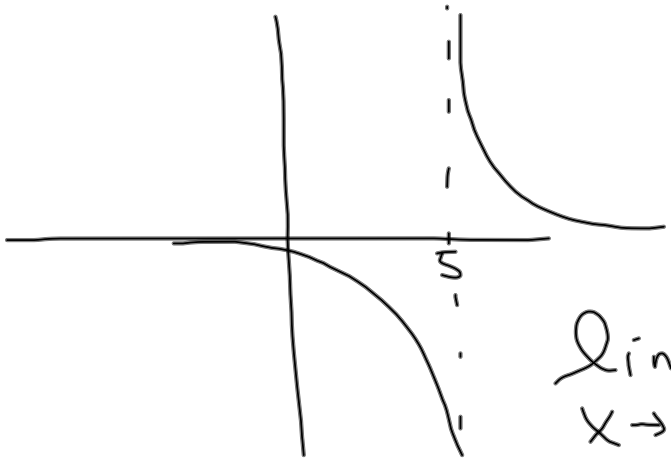
$\lim_{x \to a} f(x) = L$

means: as x approaches 'a', but is not equal to a, y approaches, or is, the real number L.

As x approaches 'a' but does not equal 'a' the function (y) values get nearer to or equal to the number L.



(∞ is not a limit... we say DNE)

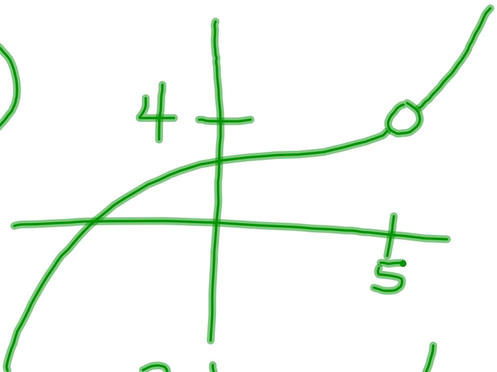


DNE

$$\lim_{x \to 5^-} f(x) = (-\infty)$$

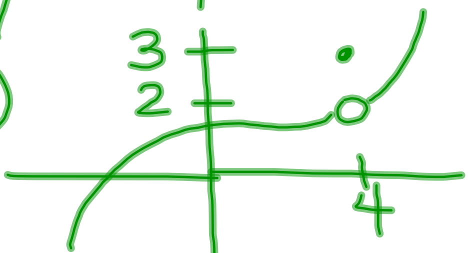
Determine the limit, if it exists:

1)



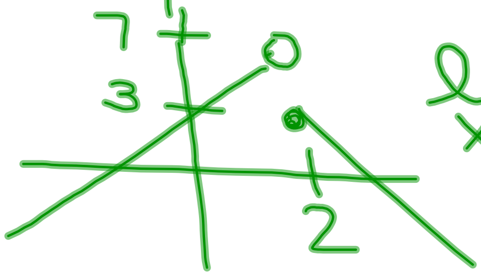
$$\lim_{x \to 5} f(x) = 4$$

2)



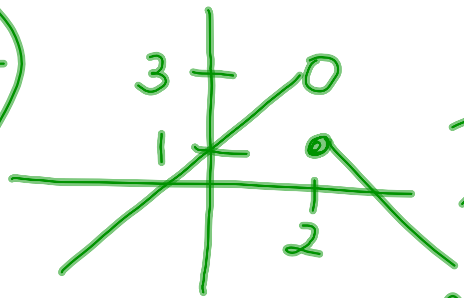
$$\lim_{x \to 4} f(x) = 2$$

3)



$$\lim_{x \to 2} f(x) = \text{DNE}$$

4)



$$\lim_{x\to 2^-} f(x) = 3$$

$$\lim_{x\to 2^+} f(x) = 1$$

$$\lim_{x\to 2} f(x) = \text{DNE}$$

Relationship Between FULL & SIDE LIMITS:

The full limit $\lim_{x\to c} g(x)$ exists iff ($\Leftrightarrow$) the side limits both exist . . . and are equal

$$\lim_{x\to c^+} g(x) = \lim_{x\to c^-} g(x) = \lim_{x\to c} g(x)$$

Indeterminate Limits

can you algebraically adjust in order to determine?

algebra to determine (usually of form $\frac{0}{0}$ or $\frac{\infty}{\infty}$)

ex $\lim_{x\to 9}\frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} = \lim_{x\to 9}\frac{x-9}{(x-9)(\sqrt{x}+3)}$

mult. top + bottom by conjugate

$\lim_{x\to 9}\frac{\cancel{(x-9)}}{\cancel{(x-9)}(\sqrt{x}+3)} = \frac{1}{6}$

Limits which require a bit of

Expand + simplify

ex $\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$

$\lim_{h\to 0}\frac{x^2+2hx+h^2-x^2}{h}$

Factor

$\lim_{h\to 0}\frac{2hx+h^2}{h} = \lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$

ex $\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}$

$\lim_{x\to 9}\frac{1}{\sqrt{x}+3} = \frac{1}{6}$

$(\sqrt{x}-3)(\sqrt{x}+3)$

get com. den.

Adjust Algebraically

ex $\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1}$

$\lim_{x\to 1}\frac{(1-x)}{x(x-1)} = \lim_{x\to 1}\frac{-1}{x} = -1$

other ways?

Graph

Numerically.

Intuitively

$\lim_{x\to\infty}\frac{2x^2+3}{4x^2+x+1}$

Rewrite Form

$\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{\frac{2x^2}{x^2}+\frac{3}{x^2}}{\frac{4x^2}{x^2}+\frac{x}{x^2}+\frac{1}{x^2}} = \frac{1}{2}$

$\frac{2x^2}{x^2}\to 2$, $\frac{3}{x^2}\to 0$, $\frac{4x^2}{x^2}\to 4$, $\frac{x}{x^2}\to 0$, $\frac{1}{x^2}\to 0$

C. LUCAS---CALCULUS
MAX/MIN DEFINITIONS

Generally speaking, there is more than one way to define a concept in mathematics. Most ideas can be discussed intuitively as well as using rather "tight" mathematical language (which includes the use of symbols). The advantage to defining a concept using the mathematical language is that there is little room for negotiation; the language is very precise. However, while a student is learning mathematics, the use of symbols and mathematical language can trigger roadblocks since the language has not become second nature yet, and an intuitive, or less precise definition, may be preferable in order to deepen comprehension. Understand, however, that while you may understand an intuitively defined concept better, it may not encompass the exceptions, and may leave some room for misinterpretation. (This is the mathematician's biggest argument against using or writing intuitive definitions, unfortunately).
Your best bet as a student is to first grasp at an intuitive understanding, and then reconcile the mathematical definition using your intuitive definition. This will help you develop a greater understanding and appreciation of the mathematical language, and enable you to be fluent in it in the future. (When you take upper level mathematical courses in which only mathematical definitions are provided, you then will be able to translate for yourself into the intuitive realm!)

Here is an example of a concept that can be defined both mathematically and intuitively!

LOCAL (or RELATIVE) MAXIMUM:

Intuitive: *The largest function value in a neighborhood of others (to the left and to the right).*
(Do you see how this is somewhat "loose"? Different words could be changed to enhance or tighten the definition. Does it help you understand the general idea of local maximum?)

Mathematical: *Let m be an interior point of the domain of the function f. Then, $f(m)$ is a local maximum for f if and only if $f(x) \le f(m)$ for all x in some open interval containing m.*
(Do you see how this covers exceptions? Can a local maximum be the same value as the function value next to it? Are we talking x values or y values?)

Now what was that intuitive definition of a limit we discussed earlier?

2) Define, as best you can what a limit is. That is, $\lim_{x \to a} f(x) = L$ means:

that as x *approaches*, but is not equal to 'a', function values (f(x)) approach, or are equal to the number 'L'.

There is a mathematical definition of it you should at least SEE in your calculus course.

Mathematical Definition

$$\lim_{x \to a} f(x) = L$$

means: