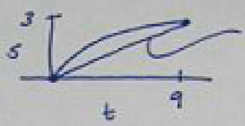
SOME review problem answers

1) $\dfrac{\sqrt{9}-\sqrt{0}}{9-0} = \dfrac{3}{9} = \dfrac{1}{3}$



Average velocity is slope $\dfrac{\text{change in position}}{\text{change in time}}$

2) $\dfrac{\sqrt{4.1}-\sqrt{3.9}}{4.1-3.9} \approx s'(4) \approx .25001 \leftarrow$ (approx velocity at 4 sec)

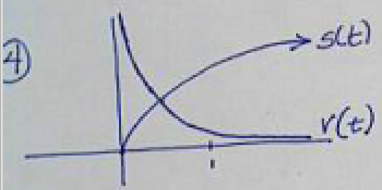
3) $s'(t) = \lim_{h\to 0} \dfrac{s(t+h)-s(t)}{h}$ ← note

$s'(t) = v(t) = \lim_{h\to 0} \dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

$= \lim_{h\to 0} \dfrac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$= \lim_{h\to 0} \dfrac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \dfrac{1}{\sqrt{x+h}+\sqrt{x}} = \dfrac{1}{2\sqrt{x}}$

4)



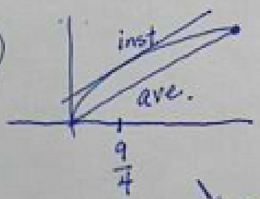
5) Check: $y_1 = \sqrt{x}$ $y_2 = \text{der1}(y_1, x)$ Looks similar to answer for #4

The position is always increasing, but at a decreasing rate (velocity is decreasing)

Particle is slowing down (p.s. $a(t) < 0$ $v(t) > 0$ ∴ slowing down)

6) $s''(t)$ is acceleration ... it would be neg. here (since $v(t)$ is decreasing) so the graph would be below the x-axis.

7)



yes! We found the slope of the chord to be $\frac{1}{3}$

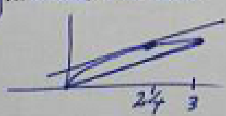
We want the slope of the tangent to be $\frac{1}{3}$ too

$\dfrac{1}{2\sqrt{x}} = \dfrac{1}{3}$

$\dfrac{3}{2} = \sqrt{x}$ $x = \dfrac{9}{4}$

$\left(c = \dfrac{9}{4}\right)$

actually... would look more like

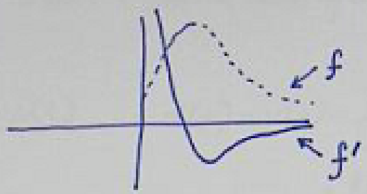


8) See notes... how slope of chord is adjusted (by moving point closer) to get slope of tangent.

(9) No, not all continuous functions are differentiable... $\sqrt[3]{x}$ at $x=0$ has vertical tange[nt].
$\sqrt{x}$ at $x=0$ doesn't have left hand derivative
$|x|$ at $x=0$ has different left an[d] right hand derivatives.

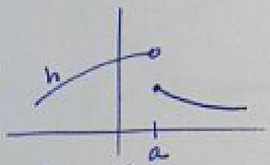
(10) $y_1 = \dfrac{2x}{\sqrt{x^3-x^2+x}}$

$y_2 = \text{der1}(y_1, x)$



(11) $\displaystyle\lim_{x\to -2} \frac{x^3+8}{x^4-16} = \lim_{x\to -2} \frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{(x+2)}(x-2)(x^2+4)} = \frac{12}{-32} = \frac{-3}{8}$ Check graphically or numerically!

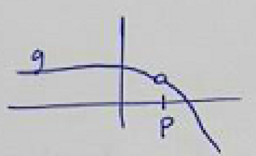
(12)



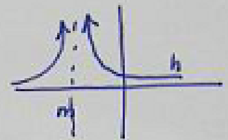
$\lim_{x\to a} h(x)$ does not exist

watch your notation!

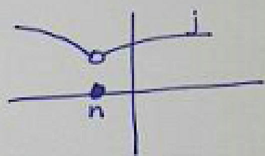
Use the correct/appropriate letters



$g(p)$ does not exist

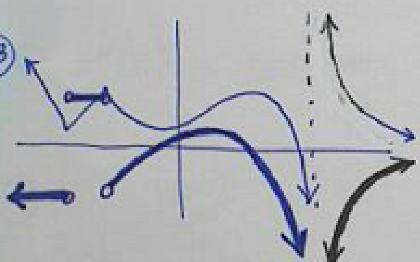


$\lim_{x\to m} h(x)$ does not exist



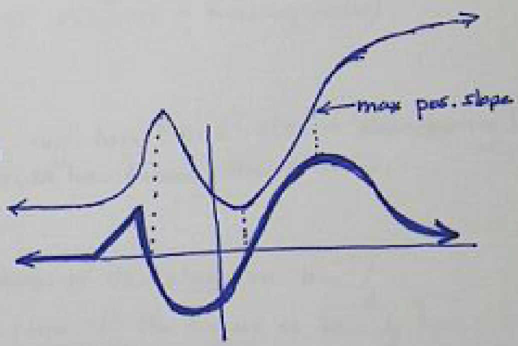
$\lim_{x\to n} j(x) \neq j(n)$

(13)



Bold is derivative.

(14)



(15) First we need the slope (we could approximate it, find it using limits, ... or remember that the derivative of x^2 is $2x$)

$\displaystyle\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{x^2-a^2}{x-a} = \lim_{x\to a} x+a = 2a$ ... so, slope at $a=3$ is 6

point is $(3,9)$ slope is 6... equation is $y = 9 + 6(x-3)$

(17) If $f''(x) < 0$ then the graph of f is concave down at x.

Graphically... look to see where the graph of f changes from concave up to concave do[wn] or vice versa.

... find a local max or min on the graph of f'

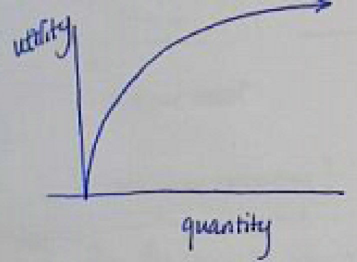
... find a root where f'' switches sign (to the right or left)

This is how one would do it algebraically... find a root of f'' and then test a point on either side to observe a sign change — if no sign change, no point of inflection.

f'' changes from pos to neg means f' goes from inc to dec means f has a point of inf.

neg to p[os] dec to in[c]

(19) a)



b) $U'(q)$ is positive, $U''(q)$ is negative } where $U(q)$ is utility as a function of quantity

(20)

1972	1976	1980	1985
586,760	988,267	1,297,606	1,328,570

a) $\frac{dA}{dt}$: 100376.75 abortions/yr; 77334.75 abortions/yr; 6192.8 abortions/yr

b) $\frac{dA}{dt}$ appears to be decreasing ∴ $\frac{d^2A}{dt^2}$ would be negative

c) # Abortions in 1989 ≈ # Abortions in 1985 + Rate of change around 1985 (1989 - 1985)

$1,328,570 + 6192.8(4) = 1353341.2$