

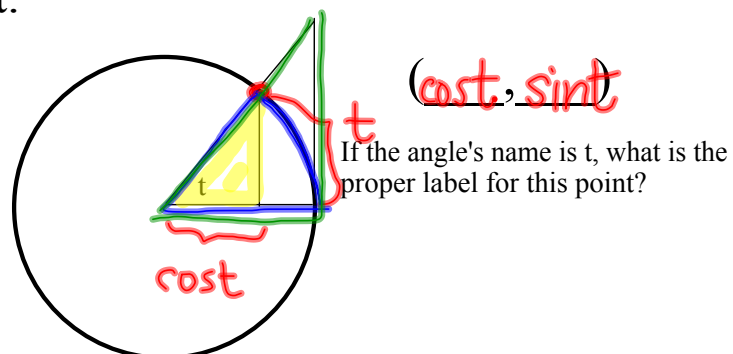
How about those Leibniz and Newton guys?

## Proof of a very important limit

(continued)

So, you have concluded that  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$  equals 1. Now, we'll prove it using the Sandwich Theorem.

Consider the unit circle and triangles and sector for one region in it.



We can compare the areas of these triangles and sector.

Areas:

$$\frac{2\pi}{\text{Perim}_\circ} = \frac{\pi}{\text{Area}_\circ}$$

$$\frac{t}{\text{Perim}_\Delta} = \frac{\frac{t}{2}}{\text{Area}_\Delta}$$

Small triangle area < Sector area < Large triangle area

$$\frac{1}{2} \text{sint cost} < \frac{t}{2} < \frac{1}{2} \cdot 1 \cdot \frac{\text{sint}}{\text{cost}}$$

$$\frac{h}{1} = \frac{\text{sint}}{\text{cost}}$$

What is the area of each region?

$$\frac{1}{2} \text{sint cost} < \frac{t}{2} < \frac{1}{2} \cdot \frac{\text{sint}}{\text{cost}}$$

$$\text{sint cost} < t < \frac{\text{sint}}{\text{cost}}$$

mult by 2

$$\text{cost} < \frac{t}{\text{sint}} < \frac{1}{\text{cost}}$$

div. by sint > 0

$$\frac{1}{\text{cost}} > \frac{t}{\text{sint}} > \text{cost}$$

$$\lim_{t \rightarrow 0} \frac{1}{\text{cost}} = 1$$

$$\lim_{t \rightarrow 0} \text{cost} = 1$$

$\therefore$  By sandwich thm  $\lim_{t \rightarrow 0} \frac{\text{sint}}{t} = 1$

**Hint:**

- easy with point
- think portion of a circle (the outer perimeter of this sector is 't')
- think similar triangles in order to get the height...you know the base.

After we do  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

How would you algebraically calculate

$$\lim_{x \rightarrow 0} \left( \frac{\cos x - 1}{x} \right) \left( \frac{\cos x + 1}{\cos x + 1} \right)$$

$$\lim_{x \rightarrow 0} \frac{\cos^2 x - 1}{x(\cos x + 1)}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\lim_{x \rightarrow 0} \frac{-\sin^2 x}{x(\cos x + 1)}$$

$$\lim_{x \rightarrow 0} \frac{-\sin x \cdot \sin x}{x(\cos x + 1)}$$

$$\lim_{x \rightarrow 0} \left[ \frac{\sin x}{x} \cdot \frac{-\sin x}{\cos x + 1} \right] = 0$$

$\downarrow$                        $\downarrow$   
1                       $\frac{0}{2}$

For tomorrow (Friday):

- Print out the Text assignment list (on Moodle)
- ~~Know how to define Continuity~~
- Read Theorem 1 on page 47
- Look over your notes and Precalc review packet
- 35 point quiz on current material Monday