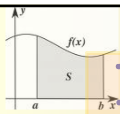
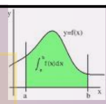
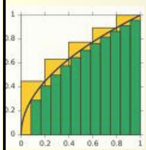
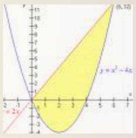
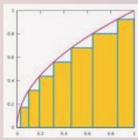
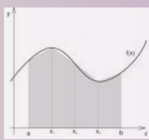
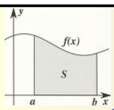
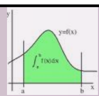



Integration Technique #3
Trig Identities

General forms we'll be integrating here:

$$\int \sin^m x\,dx$$
$$\int \sin^m x \cos^n x\,dx$$
$$\int \cos^n x\,dx$$
$$\int \tan^m x \sec^n x\,dx$$
$$\int \tan^m x\,dx$$
$$\int \sec^n x\,dx$$

Identities help make useful substitutions

Some useful identities:

Pythagorean Identities:

1) $\sin^2 x+\cos^2 x=1$

div by $\cos^2 x$
2) $\tan^2 x + 1 = \sec^2 x$

div by $\sin^2 x$
3) $1 + \cot^2 x = \csc^2 x$

Double Angle Identities:

1) $\sin(2x)=$ $2\sin x\cos x$

2) $\cos(2x)=$

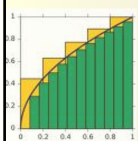
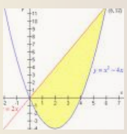
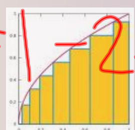
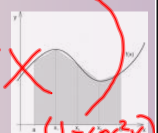
$= \cos^2 x - \sin^2 x$

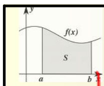
$(1-\sin^2 x)$

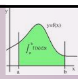
$= 1 - 2\sin^2 x$

$(1-\cos^2 x)$

$= 2\cos^2 x - 1$





Half angle identities

come from $\cos 2x$ identities

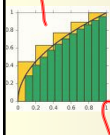
$$\cos(2x) = 1 - 2\sin^2 x$$

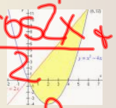
solve for $\sin^2 x$

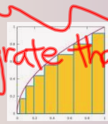
$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$\int \sin^2 x\,dx$

Whenever you are asked to integrate solely an even power of $\sin x$ $\left\{ \int \sin^2 x\,dx \quad \int \sin^4 x\,dx \right.$ replace $\sin^2 x$ with $\frac{1-\cos 2x}{2}$ & integrate that instead!



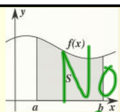


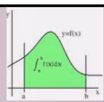


$$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx$$

$$= \frac{1}{2}\int 1 - \cos 2x\,dx$$

$$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + C$$





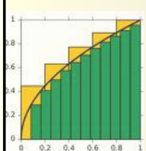
Now we need a replacement for $\cos^2 x$...

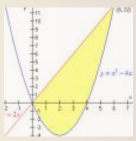
$$\cos 2x = 2\cos^2 x - 1$$

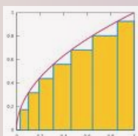
solve for $\cos^2 x$...

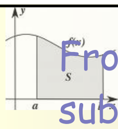
$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

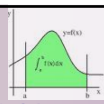
replace $\cos^2 x$ when it solely appears in integral with







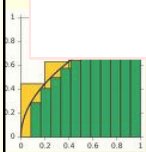


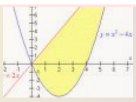


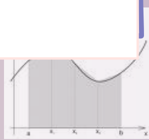
From the cos(2x) identities, we can make substitutions for $\cos^2 x$ and $\sin^2 x$. These are important to know!

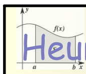
$\sin^2 x =$

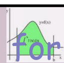
$\cos^2 x =$











Heuristic: Exploit "twin pairs" for substitution. Try to obtain

$$\int f(\text{trig func})(\text{its twin deriv})$$

split off a deriv. & convert rest of expression (using identities) to the function whose derivative you split off.

Examples

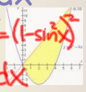
1) $\int \sin^3 x\,dx = \int \underline{\sin^2 x} \cdot \underline{\sin x\,dx} = \int (1-\cos^2 x)\sin x\,dx$

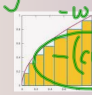
convert to cos x exp.

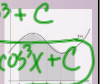
$w = \cos x$
$dw = -\sin x\,dx$

$-\int (1-w^2)\,dw =$
$-w + \frac{1}{3}w^3 + C$
$-(\cos x) + \frac{1}{3}\cos^3 x + C$

2) $\int \cos^5 x\,dx$







$(\cos^2 x)^2 = (1-\sin^2 x)^2$

$\int \underline{\cos^4 x}\,\cos x\,dx$

convert to sin x

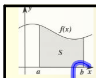
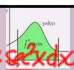
$\int (1-\sin^2 x)^2 \cos x\,dx$

Let $w = \sin x$
$dw = \cos x\,dx$

$\int (1-w^2)^2\,dw = \int (1-2w^2+w^4)\,dw$

$= w - \frac{2}{3}w^3 + \frac{1}{5}w^5 + C$

$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$

3) $\int \tan^5 x \sec^4 x \, dx$

$\int \tan^5 x \sec^2 x \sec^2 x \, dx$

convertible to tanx

deriv of tanx

$\int \tan^5 x (\tan^2 x + 1) \sec^2 x \, dx$

$w = \tan x$

$dw = \sec^2 x \, dx$

$\int w^5 (w^2+1) \, dw = \int w^7 + w^5 \, dw$

$\frac{1}{8}\tan^8 x + \frac{1}{6}\tan^6 x + C$

$\int \tan^4 x \sec^3 x \sec x \tan x \, dx$

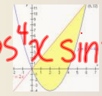
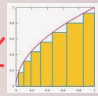
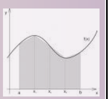
convertible to secx

deriv. of secx

(Both work)

4) $\int \sin^3 x \cos^4 x \, dx$

$\int \sin^2 x \cos^4 x \sin x \, dx$

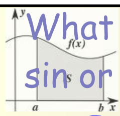
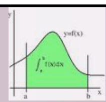
$\int (1-\cos^2 x)\cos^4 x \sin x \, dx$

$w = \cos x$

$dw = -\sin x \, dx$

$-\int (1-w^2) w^4 \, dw$

easy

What do you do if the power on sin or cos is even?

5) $\int \cos^2 x \, dx$

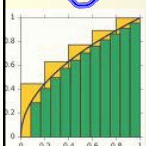
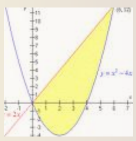
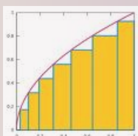
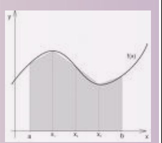
half angle!

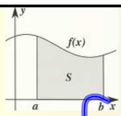
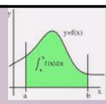
$$\int \frac{1+\cos 2x}{2} \, dx = \frac{1}{2}\int (1+\cos 2x) \, dx$$

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + C$$

eh

6) $\int \sin(4x)\cos(3x)dx = \int (1/2)[\sin(7x)+\sin(x)]dx$

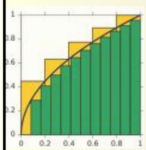
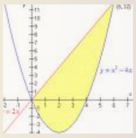
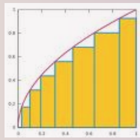
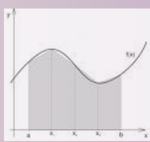



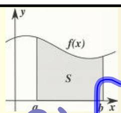
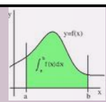
7) $\int \sin^2 x \cos^2 x \, dx$

$$\int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx$$

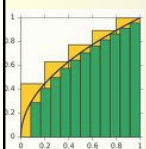
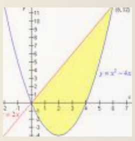
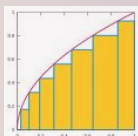
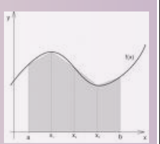
$$\frac{1}{4}\int \left(1 - \underbrace{\cos^2 2x}\right) dx$$

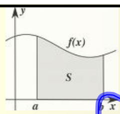
$$\frac{1}{4}\int 1 - \left[\frac{1+\cos 4x}{2}\right] dx$$

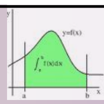





8) $\int \cos^4 x \, dx$





$$9)\ \int \sec^3 x\,dx = \sec x\tan x - \underbrace{\int \tan x\cdot \sec x\tan x\,dx}$$

$u = \sec x$

$dv = \sec^2 x\,dx$

$v = \tan x$

$du = \sec x\tan x\,dx$

$$-\int \sec x\tan^2 x\,dx$$

$$-\int \sec x(\sec^2 x - 1)\,dx$$

$$-\int \sec^3 x - \sec x\,dx$$

$$\int \sec^3 x\,dx = \sec x\tan x + \int \sec x\,dx - \int \sec^3 x\,dx$$

$$2\int \sec^3 x\,dx = \sec x\tan x + \ln|\sec x + \tan x| + C$$